THE ATTACK OF A TARGET WITH THE SIMULTANEOUS USE OF AIR AND ARTILLERY

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THESIS

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ABSTRACT

The purpose of this report is to assess the feasibility of attacking a target with the simultaneous use of air and artillery. A method for generating circular error probability as a function of release altitude is presented. Techniques for determining probabilities of kill for the air attack system, artillery system, and for the combined air-artillery attack system are examined. From the probability of kill information and from the rate of fire (delivery) of the systems, expected time to target destruction calculations are developed. The restrictions that allow the use of the combined air-artillery attack system are presented, as well as a discussion of the advantages and disadvantages of this system of attack.

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I. INTRODUCTION

Throughout the course of history there has been an interaction between technology and military tactics. For example in Europe prior to 1330 the use of the mounted armored knight or cavalry had relegated the infantry or foot soldiers to a relatively minor role in battle. In the battle of Bannockburn in 1332 the Scots introduced dismounted knights who fought as mailed spearmen defending the archers (using the long bow) on both wings. When the attacking English had been demoralized by the long bow's arrows the knights took to horse again in their proper cavalry function of charging the scattered English force. This battle served to demonstrate the value of a balanced force and the value of the proper use of the technology (the long bow) of the time. (Ref. 14)

Recent developments in military hardware make possible an interesting and potentially effective tactic, the attack of a target with the simultaneous use of air and artillery in close support of friendly forces. This tactic represents a significant change in doctrine. The hardware developments include, 1) automated unit position location and reporting system data link (Ref. 12), 2) high accuracy target locating and ranging devices (lasar designators and ranging devices, anti-personnel radars, plus other passive and active sensors) (Ref. 12), 3) rapid fire, mobile, multibarrel, radar/optically

directed anti-aircraft machine gun systems (12.7mm, 14.5mm, 23mm, 57mm, 20mm Vulcan), plus hand launched anti-air missiles (strella, redeye, etc.) (Ref. 12), 4) compact high accuracy (8-12 mils) air delivery systems (based upon inertial platforms, multimode radars, digital bombing computers, AN/TPQ-27 radar, etc.) (Ref 20), and 5) the new concept of the universal forward observer who functions as a forward air controller, a naval gunfire spotter, and an artillery forward observer (Ref. 21).

The purpose of this report is to investigate the practicability of attacking a target with the simultaneous use of air and artillery in close support of friendly forces. It will be necessary to consider the restrictions that must be imposed on the air delivery system (minimum altitudes, azimuth of attack, etc.) and on the artillery system (low angle fire, no VT fuzes, etc.). These restrictions will allow discussion of the advantages and disadvantages of the method of attack.

The infantry commander has a wide variety of options available to him with respect to the use of supporting arms. Upon encountering an enemy force the infantry commander must decide how he will attack the enemy unit. The commander consults his operations officer, air liaison, and artillery liaison officers, if the situation permits, and formulates a plan of attack. If it is decided that close air support is required the commander must request the air support from higher headquarters (the Direct Air Support Center in U.S.M.C.

tactical organizations) unless he is fortunate in having air on-station (Ref. 9). The request for close air support is processed by higher headquarters to the squadron that scrambles the air strike. A minimum of 15 to 30 minutes is required between the request for air support and the arrival of the strike force on station. Due to the mobility of many targets such as mechanized infantry or armor, it is often necessary to "fix" or pin down the enemy force in the target area. If the enemy force is not pinned down by some means prior to the arrival of the attack aircraft, the enemy force may not be visible or may be too close to friendly forces to allow the air to effectively attack. To prevent the enemy force from leaving the battle area and in an effort to cause enemy casualties the commander can use his supporting mortars and artillery. Artillery and mortar fire can begin within one to five minutes, if immediate clearance to fire is obtained from the Fire Support Coordination Center. Upon arrival of the attack aircraft on-station the artillery and mortar units stop firing. The forward air controller orients the attack aircraft with respect to the target location, friendly unit location, azimuth of attack, and mode of delivery of ordnance. One or more non-firing passes may be required to insure proper target identification. This procedure results in a two to eight minute delay between the last impact of an artillery or mortar round and the first air ordnance effect on the target area. It is during this two to eight minute delay that the enemy force has the

opportunity to prepare anti-aircraft weapons or alternatively to disperse and vacate the target area.

Studies have shown that there is little that can be done to diminish the 15 to 30 minute reaction time between request for air support and arrival of the attack aircraft. The problem associated with the delay in arrival of close air support is reduced through the use of artillery and mortars. The communication of target location and description from a ground vantage to the aircraft viewpoint is at best difficult. The attack aircraft have the additional problem of locating and identifying friendly forces near the target area. In close air support strikes one aircraft attacks at a time, hence only a small portion of the target area is actually under fire at any time. The intermittent nature of fire from air strikes and the small area that actually receives fire can have a negative effect in terms of aircraft losses and in terms of reduced bombing accuracy due to enemy ground fire.

At this point it is reasonable to ask, "Why do we interrupt the artillery fire on the target area, when this fire can reduce or eliminate the enemy ground fire and can also cause enemy casualties?" Instead of continuous fire on the target area, current doctrine results in intermittent fire from one aircraft at a time on a small portion of the target area. Borrowing from the infantry tactic of the base of fire, we could use the artillery like the base of fire and the attack aircraft as the maneuverable attacking force.

The artillery fire could be concentrated against the enemy anti-air systems and also against the main hard target which is the prime target of the attack aircraft. With the enemy anti-air systems under attack by the artillery, the close air support aircraft could press their attack without having to be overly concerned about the enemy anti-air systems. By using both air and artillery at the same time we now have continuous fire over the whole target area.

One consideration that is currently given for not using artillery and air on the same target is that aircraft and artillery shells in the same air space at the same time result in very nervous pilots, poor bombing accuracy, and high aircraft losses. An analysis in this paper will be made with respect to the extent, if any, to which aircraft and artillery shells will compete for air space in attacking the same target at the same time.

Air system models are described in Chapter II. Included in the air models are CEP-Altitude of Release relationships, slant range model, probability of kill calculations, radius of turn, and expected number of bombs to destroy a target. Chapter III is devoted to the development of the artillery models, parameters, and ballistic characteristics. 155mm Howitzer data is used to generate the required input parameters for the models. In Chapter IV the interaction between the artillery projectiles and the attack aircraft is modelled, plus the concept of the danger space is developed. The air attack and artillery system restrictions, summary of results, conclusions, and recommended topics for further investigation conclude this report.

II. THE AIR DELIVERY MODEL

As an integral part of this investigation it is useful to develop a relationship between circular error probability and altitude of weapon release for various attack aircraft. This relationship will be used as a source of input data for probability of kill calculations, expected number of bombs to kill, and other calculations. A brief overview of current air system characteristics and other factors which contribute to circular error probability will precede the development of the air models.

Current air systems developments indicate that it is reasonable to assume that the attack aircraft of the mid-to-late 1970's will be at least as effective in firing accuracy as the currently flying A-7E and A-6E aircraft. The A-7E and A-6E systems include an inertial platform, a multimode radar system, a heads-up display, a very flexible digital bombing computer, and other sensors (Ref. 6, 7, 15, 19, 20). The digital computer may be used to compute lead angles for aiming rockets and machine guns, and for determining the best release point for delivering ballistic and terminally guided ordnance. The avionics system allows both manual and automatic release of bombs and rockets and firing of machine guns (Ref. 15, 20). The avionics of these two attack aircraft allow a great deal of flexibility in the manner in which air ordnance is delivered and also in the type of ordnance which they may use.

Errors that affect an air-delivered weapon after release from the aircraft are called inherent weapon delivery errors. These inherent weapon delivery errors do not include sensor errors (elevation angle, azimuth, range, 3-axis velocity measurement), computer errors (ballistic equation fit and release time), release errors (ejection velocity and release delays), crew errors (cursor position and pilot error), and other alignment errors prior to the release of the weapon. Even if all errors prior to the release of the weapon were negligible, the inherent errors would cause the weapon to have a significant circular error probability. Inherent errors are due to weapon dispursion, non-uniform atmospheric density, and wind variations between the launch point and the target.

Ballistic dispersion or ballistic error is primarily a function of weapon design and manufacturing quality control. This error is proportional to the slant range from the aircraft at the time of release to the point of impact. Actual Ballistic Dispersion data for weapons currently in use by U.S. forces are available from sources at the Naval Weapons Center, China Lake, California (Ref. 3). For the purposes of this study, ballistic dispersion will be given a value of 5 mils.

Errors due to wind are directly proportional to the time of flight of the weapon. The time of flight depends upon aircraft velocity, dive angle, and altitude at time of release of the weapon. There are three categories of wind

error; profile, shear, and gust. Profile wind error has the greatest effect on ballistic dispersion while gust error has the least. Profile wind has a steady-state horizontal wind velocity which consists of two independent orthogonal components whose magnitudes are random variables. An avionics system such as the A-6E and A-7E system with an inertial platform is capable of effectively negating the effects of profile wind error. Shear wind error is caused by variations between the release altitude and the target. To compensate for this error it is necessary to determine the wind variations between the release altitude and the target. Wind gust errors are random and unpredictable within any micro-atmospheric structure.

System and ballistic errors are measured in mils. This means that for every 1000 units of path length that an expected error of one unit/mil is realized. Due to the relatively small cross-sectional area, the aerodynamic design, and the high density of low drag ballistic weapons it will be assumed that the free flight path of a low drag bomb is essentially frictionless.

Slant range is measured along the flight path of a ballistically dropped bomb from point of release to point of impact. The trajectory of a low drag bomb is pictured in Figure 1.

Calculations of slant range will be based upon several assumptions: a 450 knot initial velocity at time of release, a one-G release, a known altitude of release, and a known dive

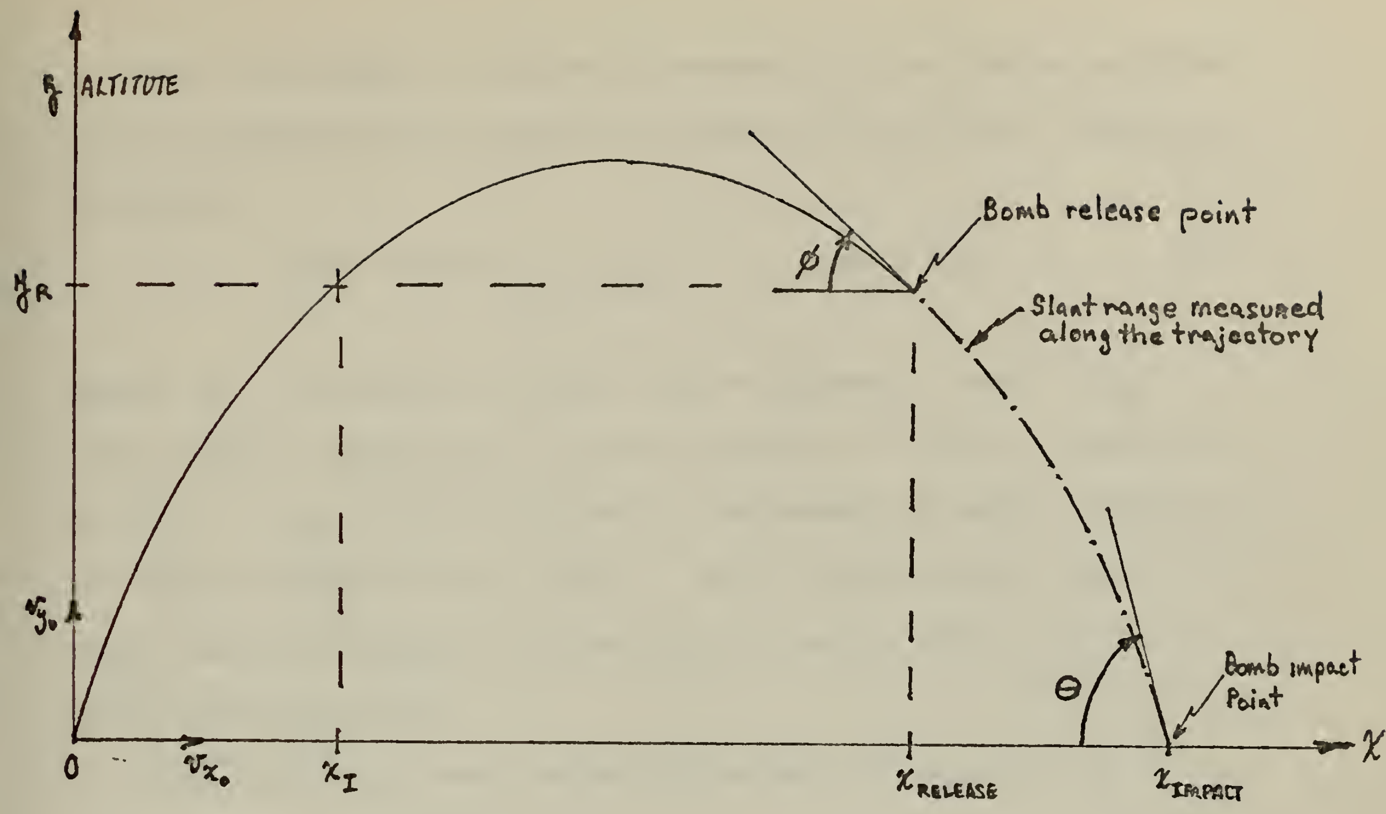


Figure 1. The Trajectory of a Low Drag Bomb (from $x_{RELEASE}$ to x_{IMPACT})

angle. From the physics of a frictionless free-falling body it is possible to derive a function for the trajectory such as that shown in Figure 1. The time of flight of the bomb is given by,

$$T = \frac{\sqrt{v_R^2 \sin^2 \phi + 2y_R g} - v_R \sin\phi}{g} \quad , \text{ (Ref. 16)} \qquad (1)$$

where v_R = release velocity in feet/second,

ϕ = dive angle of the aircraft at time of release,

y_R = release altitude in feet,

g = accelleration of gravity, a constant, = 32.16ft/sec^2,

T = time of flight in seconds.

Once the time of flight is known it is a simple calculation to determine the angle of impact of the bomb, which is given by,

$$\theta_o = \arctan(v_{y_o}/v_{x_o}) \;, \qquad \text{(Ref. 16)} \qquad (2)$$

where $v_{y_o} = v_R \sin\emptyset + g \cdot T$ and $v_{x_o} = v_R \cos\emptyset$, where v_{y_o} = the vertical component of bomb velocity at time of impact in ft./sec., v_{x_o} = the horizontal component of bomb velocity at time of impact in ft./sec., θ_o = the angle of impact of the bomb with respect to the ground, and g and T are as previously defined.

The low drag bomb trajectory shown in Figure 1 is described by,

$$y = (v_{y_o}/v_{x_o})x - (g/2v_{x_o}^{\ 2})x^2. \quad \text{(Ref. 16)} \qquad (3)$$

Table 1 contains a summary of data generated from the solution of equations (1) and (2). This data will be used in solving the slant range and circular error probability equations.

Table 1. Slant Range and CEP Input Data

BASIS: 450 KNOT A/C SPEED, 1-g. RELEASE

30° DIVE			45° DIVE		
Altitude (Feet)	Time of flight (Seconds)	Impact Angle (Degrees)	Altitude (Feet)	Time of flight (Seconds)	Impact Angle (Degrees)
3,000	6.247	41.57	3,000	4.854	52.23
5,000	9.414	46.06	5,000	7.586	55.48
7,000	12.166	49.53	7,000	10.024	58.00
9,000	14.633	52.28	9,000	12.258	60.03
10,000	15.784	53.45	10,000	13.312	60.91

A theorem from calculus states, "If $y = f(x)$ is the equation of a given curve, and if $y' = f'(x)$ is a continuous function of x, the arc of the curve from $x = \alpha$ to $x = \beta$ is rectifiable, and its length S is given by:

$$S = \int_{\alpha}^{\beta} \sqrt{1 + [f'(x)]^2}\, dx \text{ ."} \quad \text{(Ref. 17)}$$

The slant range of a low drag bomb may be calculated by applying the above theorem to equation (3) with appropriate limits. For simplification let $a = v_{y_o}$ and $b = -(g/2v_{x_o}{}^2)$. Equation (3) becomes, $y = ax + bx^2$, and $dy/dx = y' = a + 2bx$. Slant range, S, which is measured in feet, is given by:

$$S = \int_{x_{RELEASE}}^{x_{IMPACT}} (1 + a^2 + 4abx + 4b^2x^2)^{\frac{1}{2}}\, dx. \qquad (4)$$

By symmetry (Figure 1) it is apparent that the arc length of equation (3) from $x = 0$ to $x = x_I$ is the same as the arc length from $x = x_{RELEASE}$ to $x = x_{IMPACT}$.

Standoff range, which is the horizontal range from point of release to point of impact of the bomb, is given by;

$$x_I = v_{x_o} \cdot T \qquad (5)$$

where v_{x_o} and T are as previously defined. Slant range is determined by solving the following equation;

$$S = \int_0^{x_I} \sqrt{A+Bx+Cx^2}\, dx, \text{ where } A = 1+a^2,\ B = 4ab, \text{ and } c = 4b^2,$$

which yeilds the following:

$$S = \left[\frac{(2Cx+B)\sqrt{A+Bx+Cx^2}}{4C} + \frac{4AC+B^2}{BC\sqrt{C}} \ln\left(\sqrt{A+Bx+Cx^2} + Cx + \frac{B}{2\sqrt{C}}\right) \right]_{x=0}^{x=x_I} \qquad (6)$$

Solution of equation (6) with input data from Table 1 is summarized in Table 2. The data contained in table 2 will be used in the determination of circular error probability as a function of release altitude.

Table 2

Slant Range, Standoff Range, Impact Angle and Time of Flight

BASIS: 450 KNOT A/C SPEED, 1-g RELEASE, 5 MIL BOMB

Altitude (Feet)	Slant Range (Feet)	Time of Flight (Seconds)	Impact Angle (Degrees)	Standoff Range (Feet)
		30° DIVE		
3,000	5,100	6.247	41.57	4,110
5,000	7,980	9.414	46.06	6,190
7,000	10,680	12.166	49.33	8,000
9,000	13,300	14.633	52.28	9,620
10,000	14,500	15.784	53.45	10,380
		45° DIVE		
3,000	3,970	4.854	52.23	2,610
5,000	6,460	7.586	55.48	4,070
7,000	8,860	10.024	58.00	5,380
9,000	11,180	12.258	60.03	6,580
10,000	12,330	13.312	60.91	7,150

Circular error probability, as formulated in this report, results from three sources; system error, ballistic dispersion, and a wind error term. System error and ballistic dispersion may be combined in a root-mean-square error function given by:

$$RMS = \sqrt{(SE)^2 + (BD)^2}\,, \tag{7}$$

where RMS = root mean square error due to the combined effects of ballistic dispersion and system error, in mils,

SE = system error, in mils,

BD = ballistic dispersion, in mils.

It must be remembered that system error and ballistic dispersion are both measured relative to the line of fall of the low drag bomb (slant range functions). To compute a circular error probability measured with respect to the ground plane an adjustment must be made for the angle of impact. Figure 2 is a diagram of the geometric considerations that are involved.

The combined system error and ballistic dispersion, RMS, is converted to ground circular error probability by;

$$CEP = \frac{(RMS)\ (SR)}{\operatorname{Sin} \theta_o}\,, \tag{8}$$

where CEP = circular error probability measured in feet,

RMS = root mean square error given by equation (7) in mils,

θ_o = angle of impact of the bomb as given by equation (2)

SR = slant range S as given by equation (6), in k-feet.

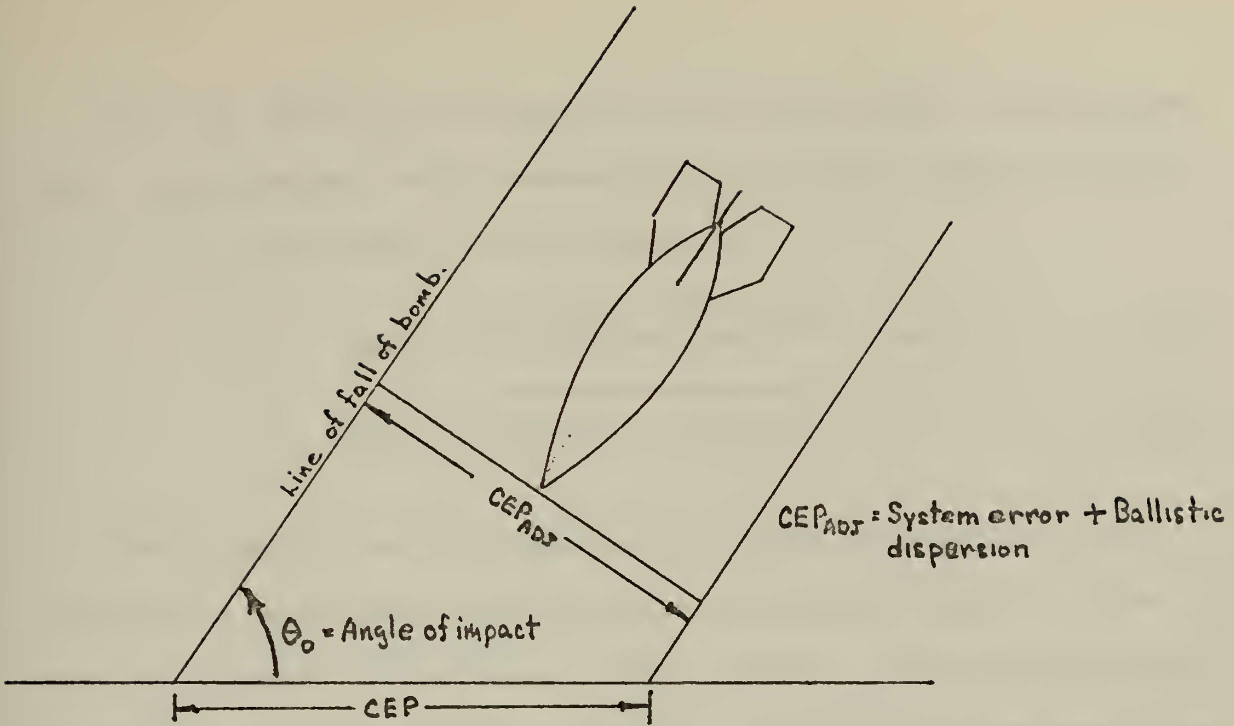


Figure 2. Ground CEP vs. System and Ballistic CEP

The circular error probability as given by equation (8) can be used to describe a system that is capable of compensating for all wind errors (except gust error).

Two wind error functions will be used to modify the CEP determined from equation (8). These functions are;

$$WE_I = W_I(T) \text{ , and} \tag{9}$$

$$WE_{II} = W_{II}(T)\,(ALT) \text{ ,} \tag{10}$$

where W_I = random wind error component (net) measured in ft/sec

W_{II} = random wind error component (net) in ft/sec, k-ft,

T = time of flight of the low drag bomb, in seconds,

ALT = altitude of release of the bomb in k-feet,

WE_I = wind error effect type I in feet, and

WE_{II} = wind error effect type II in feet.

The two wind error functions, (9) and (10), are combined with the circular error probability function, equation (8), in a root mean square error function:

$$CEP_I = \sqrt{(CEP)^2 + (WE_I)^2} \quad , \quad \text{and} \tag{11}$$

$$CEP_{II} = \sqrt{(CEP)^2 + (WE_{II})^2} \; . \tag{12}$$

The circular error probability given by equation (11) contains a wind error factor which is proportional to the time of flight of the bomb. This circular error probability would correspond to a system that is capable of compensating for part of the wind error factors between the target and the aircraft, e.g., the CP-741 system of the A-4F aircraft (Ref. 15). The circular error probability generated by equation (12) would be more characteristic of the less sophisticated bombing system of the F-4 in which wind error compensation is based primarily on pilot judgement.

Table 3 contains a summary of circular error probabilities generated by equations (8), (11) and (12) for many aircraft systems that are currently in use in the U.S. Navy and U.S. Marine Corps. The data contained in Table 3 will be used in the calculation of probability of kill and in the determination of the expected number of bombs required to kill a target in Chapter IV. Figure 3 is a plot of CEP vs. altitude of release for a 10 mil system (using equation (8)), a 26 mil system (using equation (11)), and a 40 mil system (using equation (12)).

Table 3

CEP vs. Altitude of Release

Basis: 450 knot aircraft speed, 1-G release, 5 mil bombs, advertized A-7E system error 10 mils, A-4F system error 26 mils, F-4B system error 40 mils, W_I = 2 knots, and W_{II} = 1.0 knots/k-feet. (Ref. 2, 13, 15, 19, 20)

10 Mil System Error

Release Altitude (feet)	CEP (feet)	CEP_I (feet)	CEP_{II} (feet)	Standoff Range (feet)
		30° DIVE		
3000	85.9	88.4	91.5	4110
5000	123.9	128.0	147.5	6190
7000	157.0	162.3	212.9	8000
9000	187.9	194.3	291.1	9630
10000	201.9	208.8	334.3	10380
		45° DIVE		
3000	56.1	58.5	61.2	2610
5000	87.6	91.3	108.5	4080
7000	116.7	121.6	166.3	5380
9000	144.3	150.1	235.6	6580
10000	157.8	164.0	274.6	7150

Table 3 (continued)

CEP vs. Altitude of Release

26 Mil System Error

Release Altitude (feet)	CEP (feet)	CEP_I (feet)	CEP_{II} (feet)	Standoff Range (feet)
		30^o DIVE		
3000	203.3	204.4	205.7	4110
5000	293.4	295.1	303.9	6190
7000	371.8	374.1	398.6	8000
9000	445.1	447.8	497.5	9630
10000	478.0	481.0	547.2	10380
		45^o DIVE		
3000	132.9	134.0	135.2	2610
5000	207.5	209.0	217.1	4080
7000	276.5	281.7	300.8	5380
9000	341.8	344.7	389.2	6580
10000	373.6	376.3	436.0	7150

Table 3 (continued)

CEP vs. Altitude of Release

40 Mil System Error

Release Altitude (feet)	CEP (feet)	CEP_I (feet)	CEP_{II} (feet)	Standoff Range (feet)
		30° DIVE		
3000	309.6	310.3	311.2	4110
5000	446.8	447.9	453.8	6190
7000	566.0	567.5	584.0	8000
9000	677.7	679.5	713.2	9630
10000	727.8	729.7	775.0	10380
		45° DIVE		
3000	202.4	203.1	203.9	2610
5000	316.0	317.0	322.4	4080
7000	421.0	422.3	437.3	5380
9000	520.3	522.0	552.6	6580
10000	568.8	570.6	611.6	7150

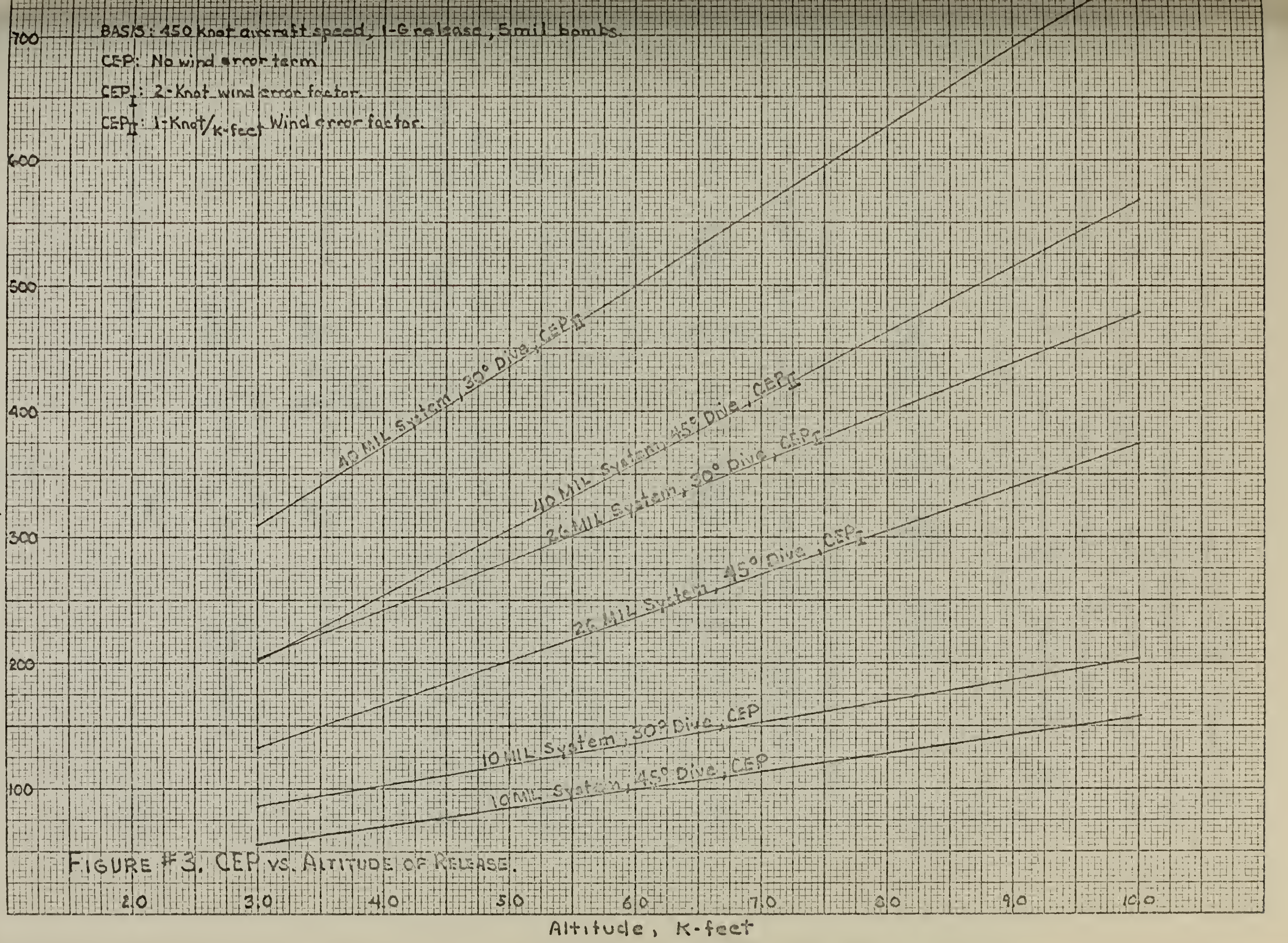


FIGURE #3. CEP VS. ALTITUDE OF RELEASE.

Implicit in the definition of circular error probability is an underlying bivariate normal distribution. It will be assumed that $\sigma_x = \sigma_x$, and that the mean point of impact of bombs dropped will be at target center. By carefully choosing x- and y-axis (thus causing ρ to equal zero) the density of this bivariate normal distribution is,

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{1}{2}\left(\frac{x^2}{\sigma_x{}^2} + \frac{y^2}{\sigma_y{}^2}\right)\right\} . \tag{13}$$

Under the assumption that the mean point of impact is at target center, the probability of hitting a target is,

$$P(\text{Hit}) = \frac{1}{2\pi\sigma_x\sigma_y} \iint_A \exp\left\{-\frac{1}{2}\left(\frac{x^2}{\sigma_x{}^2} + \frac{y^2}{\sigma_y{}^2}\right)\right\} dA , \tag{14}$$

where A = target area,

σ_x = standard deviation of impact points along x-axis,

σ_y = standard deviation of impact points along y-axis.

By assuming that the target is a circle of radius R it is possible to simplify equation (14). By letting $\sigma_x = \sigma_y = \sigma$ and converting to polar coordinates, equation (14) becomes,

$$P(\text{Hit}) = \frac{1}{2\pi\sigma^2} \int_0^{2\pi} \int_0^R \exp\left(-\frac{r^2}{2\sigma^2}\right) r \, dr \, d\theta ,$$

$$= \frac{1}{2\pi\sigma^2} \left(\int_0^{2\pi} d\theta\right) \left(\int_0^R r \exp\left(-\frac{r^2}{2\sigma^2}\right) dr\right) ,$$

which yeilds $P(\text{Hit}) = 1 - e^{-R^2/2\sigma^2}$, (15)

where $P(Hit)$ = probability of hitting the target,

R = the radius of the target, in feet,

σ = standard deviation in feet.

From the definition of circular error probability (50% of the bombs will impact within the stated value of CEP of the target) and the assumption of a bivariate normal distribution with $\sigma_x = \sigma_y$ it is possible to convert circular error probability to standard deviation by,

$$CEP = 1.1774\,\sigma_{CEP} \tag{16}$$

Suppose that in attacking a target we are interested in whether or not the target is hit on any given pass, and suppose the attack is continued until the first hit is recorded. By defining the random variable X to be the number of passes required for the first hit on the target and by assuming that each pass is independent of other passes it is possible to determine the distribution of X. Since X can only assume integer (positive) values it is apparent that k-1 failures (misses) must precede a hit on the k^{th} pass, hence,

$$P\ X{=}k = (1-p)^{k-1}\ p\ , \qquad k = 1, 2, \cdots , \tag{17}$$

is the distribution (the Geometric Distribution) of X where,

$(1-p)$ = the single pass probability of a miss, and

p = the single pass probability of a hit.

If all attack aircraft are the same type and all are dropping their bombs from the same altitude under essentially the

the same conditions, a value for p may be obtained by solving equation (15).

To determine the expected number of passes required to hit the target we may use the following relationship;

$$E(X) = \sum_{k=1}^{\infty} kp(1-p)^{k-1} = \sum_{k=1}^{\infty} kp(q)^{k-1} = p \sum_{k=1}^{\infty} \frac{d(q^k)}{dq}$$

$$= p \frac{d}{dq} \sum_{k=1}^{\infty} qk = p \frac{d}{dq} \left(\frac{q}{1-q}\right) = \frac{1}{p} , \qquad (18)$$

where E(X) = the expected number of passes required for first hit, and p is as previously described. If multiple hits on a target are required to kill the target then;

$$E(X) = \frac{1}{p \cdot p(K/H)} , \qquad (19)$$

where $p(K/H)$ = probability of kill given a target hit. Equation (19) is proved in Chapter III.

The last relationship that will be discussed in this chapter concerns the estimation of the minimum altitude that the attack aircraft will attain after dropping its bomb. The radius of turn for the attack aircraft pulling up from its dive is given by,

$$RT = \frac{v^2}{g\sqrt{N^2-1}} , \qquad (20)$$

where RT = radius of turn in feet,

v = aircraft speed in feet/second,

N = the g-loading force on the aircraft, and

g = the accelleration of gravity (32.16 ft/sec^2).

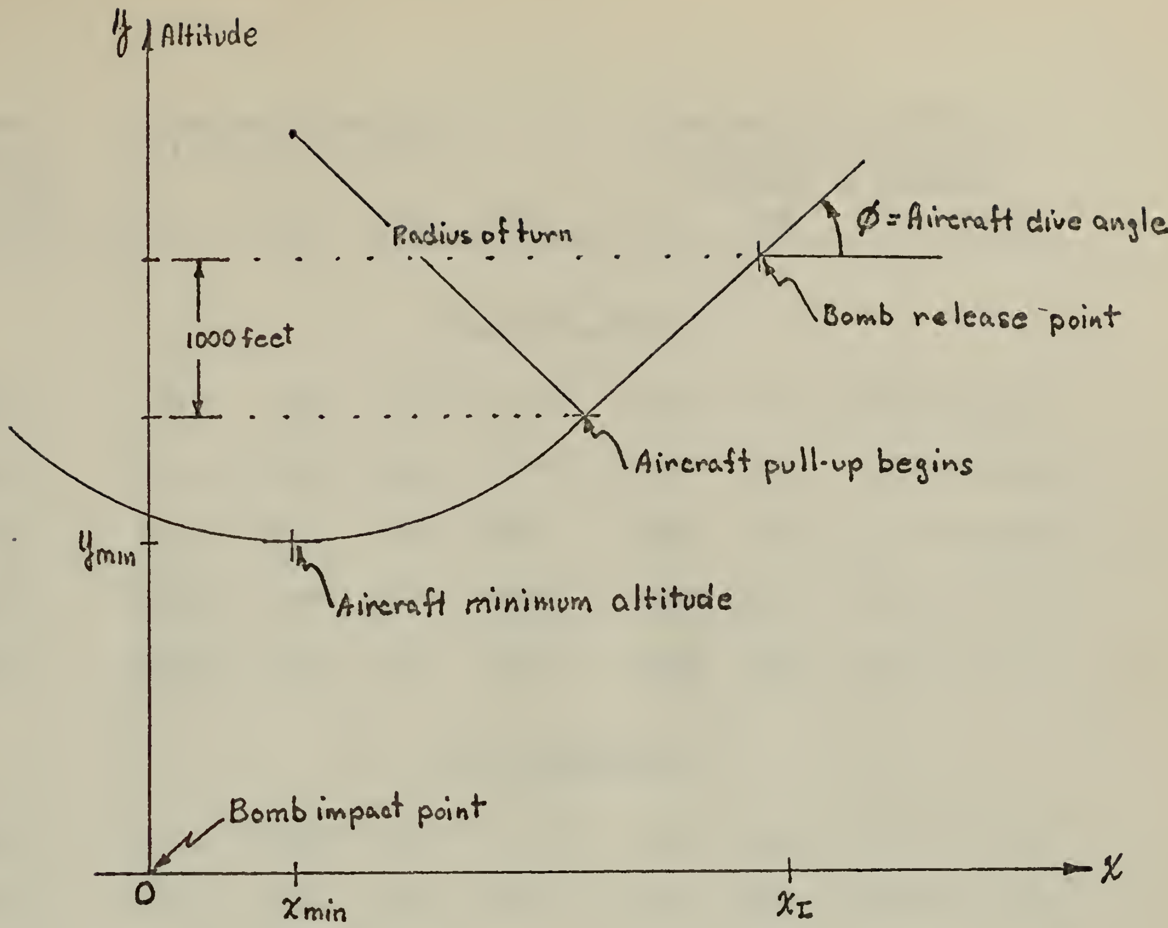


Figure 4. Geometry of Radius of Turn, Minimum Altitude and Position of Minimum Altitude.

It will be assumed that the aircraft will begin its pull up 1000 feet below the point at which the bomb is dropped. Figure 4 pictures the relevant geometric considerations.

Table 4 contains a summary of results for the solution of equation (20) and its subsequent application as per Figure 4.

Table 4. Minimum Altitude and Position of the Attack Aircraft

BASIS: 450 KNOT AIRCRAFT SPEED

G-LOADINGS	RADIUS OF TURN (FEET)
5	3,662
4	4,632
3	6,342
2	10,357

BOMB RELEASE ALTITUDE (FEET)	MINIMUM AIRCRAFT ALTITUDE IN FEET				AIRCRAFT POSITION AT MINIMUM ALTITUDE, FIGURE 4 REFERS			
	5-G	4-G	3-G	2-G	5-G	4-G	3-G	2-G
				45° DIVE DATA				
3000	930	640	140	crash	-980	-1670	-2880	crash
5000	2930	2640	2140	970	490	-200	-1410	-4250
7000	4930	4640	4140	2970	1800	1110	-100	-2940
9000	6930	6640	6140	4970	3000	2310	1100	-1740
10000	7930	7640	7140	5970	3560	2880	1670	-1170
				30° DIVE DATA				
3000	1510	1380	1160	620	280	-210	-940	-3070
5000	3510	3380	3160	2620	2360	1880	1020	-990
7000	5510	5380	5160	4620	4170	3690	2830	820
9000	7410	7380	7160	6620	5790	5310	4450	2450
10000	8510	8380	8160	7620	6550	6070	5210	3200

In this chapter we developed the formulas and relationships that will be needed to describe the air weapon system. Before considering the ramifications that are involved in attacking a target with the simultaneous use of air and artillery, it will be necessary to quantitatively characterize the artillery system. The artillery system is analyzed in Chapter III, and Chapter IV is devoted to interactions in the attack of a target with air and artillery.

III. THE ARTILLERY MODEL

Having described the air delivery system in the previous chapter, we will now focus on the artillery system. Careful attention will be directed to data dealing with maximum ordinate of trajectory, angle of impact, range error probable, and deflection error probable as obtained from the artillery firing tables currently used. Several probability of hit models will be discussed as well as methods for calculating the expected number of rounds required for the destruction of a target. The M-109, 155mm, Self-Propelled Howitzer will be used as the source of input data for the artillery models that are developed in this chapter.

Table 5 is a summary of the trajectory characteristics for the M-109, 155mm, Howitzer for low angle fire (gun elevation less than 45^{o}). Also included are data on range error probable, deflection error probable, standard deviation of range, and standard deviation of deflection. Deflection error probable and range error probable are converted to standard deviation of deflection and standard deviation of range, respectively, by application of the following conversion factors:

$$DEP = 0.6745\,\sigma_{DEF}\ , \qquad \text{(Ref. 18)} \qquad (21)$$

and $$REP = 0.6745\,\sigma_{RG}\ , \qquad \text{(Ref. 18)}$$

where DEP = deflection error probable in meters,

REP = range error probable in meters,

TABLE 5

155mm Howitzer Trajectory Characteristics

Charges: 1-5 Green, 6-7 White & 8 (Ref 4, 22)

*Data in parenthesis is upper 95^{th} percentile prediction interval data. (Ref. 22)

GUN TARGET RANGE (METERS)	MAXIMUM* ORDINATE (METERS)	ANGLE OF IMPACT, MILS (DEGREES)	STANDARD* DEVIATION RANGE (METERS)	STANDARD* DEVIATION DEFLECTION (METERS)
a) Charge 1 Green Bag				
500	7 (30)	56 (3.2)	4.4 (6.7)	0 (0.7)
1000	28 (50)	117 (6.6)	7.4 (11.1)	0 (0.7)
1500	66 (120)	181 (10.0)	10.4 (15.6)	1.5 (2.2)
2000	123 (200)	252 (14.2)	13.4 (20.0)	1.5 (2.2)
2500	204 (300)	330 (18.6)	17.8 (26.7)	1.5 (2.2)
3000	317 (440)	421 (23.6)	20.8 (31.2)	1.5 (2.2)
3500	484 (620)	533 (30.0)	25.2 (37.8)	1.5 (2.2)
4000	794 (950)	708 (39.8)	29.7 (44.6)	3.0 (4.4)
b) Charge 2 Green Bag				
500	5 (30)	44 (2.3)	5.9 (8.9)	0 (0.7)
1000	22 (60)	91 (5.1)	7.4 (11.1)	0 (0.7)
1500	52 (110)	141 (7.9)	8.9 (13.4)	1.5 (2.2)
2000	95 (180)	195 (11.0)	11.9 (17.8)	1.5 (2.2)
2500	154 (250)	254 (14.5)	14.8 (22.3)	1.5 (2.2)
3000	233 (350)	318 (17.9)	17.8 (21.7)	1.5 (2.2)
3500	337 (480)	389 (21.9)	22.8 (33.4)	3.0 (4.4)
4000	477 (640)	472 (27.5)	25.2 (37.8)	3.0 (4.4)

TABLE 5 (continued)

GUN TARGET RANGE (METERS)	MAXIMUM* ORDINATE (METERS)	ANGLE OF IMPACT, MILS (DEGREES)	STANDARD* DEVIATION RANGE (METERS)	STANDARD* DEVIATION DEFLECTION (METERS)
4500	675 (860)	575 (32.3)	29.7 (44.6)	3.0 (4.4)
5000	1074 (1270)	739 (41.5)	34.1 (51.2)	4.4 (6.7)

c) Charge 3 Green Bag

500	4 (20)	34 (1.9)	5.9 (8.9)	0 (0.7)
1000	17 (50)	70 (3.9)	7.4 (11.1)	1.5 (2.2)
1500	39 (80)	109 (6.1)	8.9 (13.3)	1.5 (2.2)
2000	72 (130)	149 (8.4)	10.4 (15.6)	1.5 (2.2)
2500	116 (190)	193 (10.9)	13.4 (20.0)	1.5 (2.2)
3000	173 (260)	239 (13.5)	14.8 (22.2)	3.0 (4.4)
3500	245 (350)	290 (16.3)	17.8 (26.7)	3.0 (4.4)
4000	336 (460)	344 (19.3)	20.8 (31.1)	3.0 (4.4)
4500	448 (580)	405 (22.8)	23.8 (35.6)	4.4 (6.7)
5000	592 (740)	474 (26.6)	28.2 (42.3)	4.4 (6.7)
5500	782 (950)	555 (31.2)	31.2 (46.7)	4.4 (6.7)
6000	1061 (1240)	661 (37.2)	35.6 (53.4)	5.9 (8.9)

d) Charge 4 Green Bag

1000	13 (40)	54 (3.0)	8.9 (13.3)	1.5 (2.2)
2000	54 (100)	113 (6.4)	10.4 (15.6)	1.5 (2.2)
3000	129 (200)	179 (10.0)	11.9 (17.8)	3.0 (4.4)
4000	243 (340)	254 (14.3)	16.3 (24.5)	4.4 (6.7)
5000	409 (530)	340 (19.1)	19.3 (28.9)	4.4 (6.7)

TABLE 5 (continued)

GUN TARGET RANGE (METERS)	MAXIMUM* ORDINATE (METERS)	ANGLE OF IMPACT, MILS (DEGREES)	STANDARD* DEVIATION RANGE (METERS)	STANDARD* DEVIATION DEFLECTION (METERS)
6000	648 (790)	441 (24.8)	25.2 (37.8)	5.9 (8.9)
7000	1010 (1180)	568 (32.0)	31.1 (46.7)	7.4 (11.1)
8000	1771 (1960)	783 (44.0)	38.6 (57.8)	8.9 (13.3)
e) Charge 5 Green Bag				
1000	10 (30)	41 (2.3)	10.4 (15.6)	0 (0.7)
2000	42 (30)	90 (5.1)	10.4 (15.6)	1.5 (2.2)
3000	102 (160)	145 (8.2)	11.9 (17.8)	1.5 (2.2)
4000	193 (270)	205 (11.5)	13.3 (20.0)	3.0 (4.4)
5000	320 (420)	271 (15.2)	16.3 (24.5)	3.0 (4.4)
6000	492 (610)	346 (19.5)	19.3 (28.9)	4.4 (6.7)
7000	724 (860)	431 (24.2)	23.7 (35.6)	4.4 (6.7)
8000	1043 (1200)	530 (29.8)	28.2 (42.3)	5.9 (8.9)
9000	1521 (1700)	656 (36.9)	34.1 (51.2)	5.9 (8.9)
9500	1900 (2090)	742 (41.7)	37.1 (55.6)	7.4 (11.1)
f) Charge 6 White Bag				
1000	6 (20)	26 (1.5)	23.7 (35.6)	0 (0.7)
2000	28 (60)	61 (3.4)	23.7 (35.6)	1.5 (2.2)
3000	70 (120)	105 (5.9)	23.7 (35.6)	1.5 (2.2)
4000	139 (200)	159 (8.9)	26.7 (40.0)	3.0 (4.4)
5000	241 (320)	218 (12.3)	29.7 (44.5)	3.0 (4.4)
6000	379 (470)	281 (15.8)	34.1 (51.2)	4.4 (6.7)

TABLE 5 (continued)

GUN TARGET RANGE (METERS)	MAXIMUM* ORDINATE (METERS)	ANGLE OF IMPACT, MILS (DEGREES)	STANDARD* DEVIATION RANGE (METERS)	STANDARD* DEVIATION DEFLECTION (METERS)
7000	558 (660)	349 (19.6)	37.1 (55.6)	4.4 (6.7)
8000	785 (810)	504 (28.3)	40.0 (60.0)	5.9 (8.9)
9000	1076 (1200)	596 (33.6)	44.5 (66.7)	5.9 (8.9)
10000	1460 (1610)	596 (33.6)	50.5 (75.7)	7.4 (11.1)
11000	2007 (2170)	708 (39.8)	56.4 (84.6)	8.9 (13.3)
12000	3225 (3410)	906 (51.0)	LARGE	11.9 (17.8)

g) Charge 7 White Bag

1000	4 (20)	18 (1.0)	35.6 (53.4)	0 (0.7)
2000	19 (40)	40 (2.3)	31.1 (46.7)	1.5 (2.2)
3000	45 (80)	69 (3.9)	29.7 (44.5)	1.5 (2.2)
4000	91 (140)	105 (5.9)	28.2 (42.3)	1.5 (2.2)
5000	158 (220)	150 (8.4)	28.2 (42.3)	3.0 (4.4)
6000	255 (330)	206 (11.6)	29.7 (44.5)	3.0 (4.4)
7000	389 (470)	268 (15.1)	32.6 (48.9)	3.0 (4.4)
8000	563 (660)	333 (18.7)	35.6 (53.4)	4.4 (6.7)
9000	781 (890)	401 (22.6)	37.1 (55.6)	4.4 (6.7)
10000	1052 (1160)	471 (26.5)	40.0 (60.0)	5.9 (8.9)
11000	1383 (1500)	545 (30.7)	43.0 (64.5)	5.9 (8.9)
12000	1796 (1930)	625 (35.2)	46.0 (68.9)	7.4 (11.1)
13000	2335 (2480)	715 (40.2)	48.9 (73.4)	7.4 (11.1)
14000	3133 (3330)	827 (46.5)	53.4 (80.1)	8.9 (13.3)

TABLE 5 (continued)

GUN TARGET RANGE (METERS)	MAXIMUM* ORDINATE (METERS)	ANGLE OF IMPACT, MILS (DEGREES)	STANDARD* DEVIATION RANGE (METERS)	STANDARD* DEVIATION DEFLECTION (METERS)
h) <u>Charge 8</u>				
1000	3 (10)	12 (0.6)	47.4 (71.2)	0 (0.7)
2000	12 (30)	26 (1.5)	43.0 (64.5)	1.5 (2.2)
3000	30 (60)	44 (2.5)	38.6 (57.8)	1.5 (2.2)
4000	59 (100)	67 (3.8)	35.6 (53.4)	1.5 (2.2)
5000	101 (150)	95 (5.4)	32.6 (48.9)	1.5 (2.2)
6000	160 (220)	130 (7.3)	31.1 (46.7)	3.0 (4.4)
7000	242 (310)	173 (9.7)	32.6 (48.9)	3.0 (4.4)
8000	353 (430)	227 (12.8)	34.1 (51.2)	3.0 (4.4)
9000	501 (590)	289 (16.3)	37.1 (55.6)	3.0 (4.4)
10000	691 (790)	355 (20.0)	41.5 (62.3)	4.4 (6.7)
11000	928 (1040)	422 (23.8)	44.5 (66.7)	4.4 (6.7)
12000	1217 (1340)	490 (27.6)	48.9 (73.4)	4.4 (6.7)
13000	1564 (1700)	559 (31.4)	51.9 (77.8)	5.9 (8.9)
14000	1978 (2130)	629 (35.4)	54.9 (82.4)	5.9 (8.9)
15000	2476 (2640)	701 (39.4)	59.4 (89.1)	5.9 (8.9)
16000	3094 (3260)	778 (43.8)	62.4 (93.6)	7.4 (11.1)
17000	3927 (4110)	867 (48.8)	68.3 (102.5)	7.4 (11.1)
18000	5736 (5940)	1023 (57.5)	74.2 (111.3)	10.4 (15.6)

σ_{DEF} = standard deviation range in meters,

σ_{RG} = standard deviation range in meters.

The upper 95th percentile prediction interval value of the data is included in Table 5 (Ref. 22). The information contained in Table 5 will be used in the interaction models that are developed in Chapter IV, as well as imput for models developed in this chapter.

It will be assumed the fall of shot over the target area is described by a bivariate normal distribution with $\rho = 0$. Then the density of the fall of shot distribution is,

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right\} , \qquad (23)$$

where σ_x = standard deviation along x-axis in meters,

μ_x = mean point of impact along x-axis in meters,

σ_y = standard deviation along y-axis in meters, and

μ_y = mean point of impact along y-axis in meters.

The probability of a hit is calculated by solving

$$P(\text{Hit}) = \iint_A \frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right\} dA , \qquad (24)$$

where A = target area in square meters. Solution of equation (24) is in general mathematically intractable except for certain target shapes. Closed-form solutions exist in the case of a circular target where it is assumed that $\sigma_x = \sigma_y$, and that the mean point of impact is at the target center as was assumed in equation (15) in Chapter II. The assumption

that $\sigma_x = \sigma_y$ is not amenable to the artillery model, therefore the result obtained in equation (15) is not applicable to artillery fire. By assuming that the target is rectangular, equation 24 reduces to

$$P(Hit) = P_x \cdot P_y \quad , \tag{25}$$

where $P_x = \int_{-L_x}^{L_x} \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp\left\{-\frac{(x-\mu_x)^2}{2\,\sigma_x^{\,2}}\right\} dx$, and

$$P_y = \int_{-L_y}^{L_y} \frac{1}{\sqrt{2\pi}\,\sigma_y} \exp\left\{-\frac{(y-\mu_y)^2}{2\,\sigma_y^{\,2}}\right\} dy \quad ,$$

where the target length along the x-axis extends from $-L_x$ to $+L_x$ and the length along the y-axis is from $-L_y$ to $+L_y$. Under the assumption of a mean point of impact at target center, P_x and P_y reduce to

$$P_x = \int_{-L_x}^{L_x} \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp\left\{-\frac{x^2}{2\sigma_x^{\,2}}\right\} dx \quad , \text{ and}$$

$$P_y = \int_{-L_y}^{L_y} \frac{1}{\sqrt{2\pi}\,\sigma_y} \exp\left\{-\frac{y^2}{2\sigma_y^{\,2}}\right\} dy \quad ,$$

which can through proper change of variables be evaluated using the standard normal distribution tables.

By assuming that the mean point of impact is at the target center, that σ_x and σ_y are "large" when compared with the target dimensions, and by proper choice of x- and y- axis

(to set $\rho = 0$), it is possible to obtain a simplified form of equation (24) to be used in determining the probability of hit. Consider the following modification to equation (24):

$$P(\text{Hit}) = \frac{1}{2\pi\sigma_x\sigma_y}\iint_A e^{-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)}\,dA$$

$$= \frac{1}{2\pi\sigma_x\sigma_y}\iint_A e^{-\left(\frac{x^2}{2\sigma_x^2}\right)}\,e^{-\left(\frac{y^2}{2\sigma_y^2}\right)}\,dA$$

$$= \frac{1}{2\pi\sigma_x\sigma_y}\iint_A \left[\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{x^2}{2\sigma_x^2}\right)^n\right]\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{y^2}{2\sigma_y^2}\right)^n\right]dA.$$

Since it has been assumed that σ_x and σ_y are "large" when compared to the target dimensions, then it can be seen that

$$\left.\frac{x^2}{2\sigma_x^2}\right|_{x=x_i} \approx 0 \quad \text{and} \quad \left.\frac{y^2}{2\sigma_y^2}\right|_{y=y_i} \approx 0$$

simultaneously for all $(x_i, y_i) \in dA$, which are points on the boundary of the target area; therefore the above integral reduces to

$$P(\text{Hit}) \approx \frac{\iint_A dA}{2\pi\sigma_x\sigma_y} = \frac{A}{2\pi\sigma_x\sigma_y}\quad, \qquad (26)$$

where A = target area in square meters,

σ_x = standard deviation range in meters, and

σ_y = standard deviation deflection in meters.

Based on the single round hit probabilities that may be calculated from equations (25) and (26), it would be desirable to determine a method for calculating the expected number of rounds required to hit a target. In his article, "On the Computation of Hit Probability," Helgert points out that for reasonable large values of round to round correlation, the assumption of statistical independance between the effects of rounds leads to large errors in the computation of the hit distribution (Ref. 11). The simplest application of the round to round correlation phenomenon involves the assumption that the correlation depends only on the immediately preceding round and no others. This assumption leads to the development of the Markov-Dependent Fire model.

Both the independence criteria and the round to round correlation criteria will be used in evaluating the expected number of rounds required to hit a target. Under the independence criteria the probability of achieving the first hit on target on the n^{th} round is given by the Geometric Distribution, $p_I(n)$ = p(achieving 1^{st} hit on n^{th} round) = $p(1-p)^{n-1}$, where p = single round hit probability from equation (25) or (26), and n = 1, 2, 3, $\cdots$. The expected number of rounds to achieve the first hit on the target may be calculated using equation (18). Under the rounds correlated criteria and using the results generated by Helgert, the probability of achieving the first hit on target on the n^{th} round is given by;

$$P_D(m) = \begin{cases} p \ , & \text{for } n=1 \\ (1-p)\left[P(m|m)\right]^{n-2} \cdot P(h|m) \ , & \text{for } n \geq 2 \end{cases} \tag{27}$$

where $P(m|m)$ = the probability of a miss given a miss on the previous round = $q + \rho p$,

$P(h|m)$ = the probability of a hit given a miss on the previous round = $p(1-\rho)$,

p = the probability that the first round is a hit which is calculated from equation (25) or (26),

q = the probability that the first round is a miss, $1-p$, and

ρ = the correlation between the $(n-1)^{st}$ round and the n^{th} round.

The expected number of rounds to first hit is given by

$$E(n) = \sum_{n=1}^{\infty} nP_D(n) = p + \sum_{n=2}^{\infty} n(1-p)\left[P(m|m)\right]^{n-2} \cdot P(h|m) \ ,$$

which after much algebraic manipulation is reduced to

$$E(n) = \frac{1-\rho p}{p(1-\rho)} \ . \tag{28}$$

Note that equation (28) reduces to equation (18) when there is independence between rounds ($\rho=0$).

Having developed a method for determining the number of rounds required to first hit we must recognize the fact that a target hit does not necessarily result in the distruction of the target. It will be assumed that the number of hits required to destroy a target is given by the Geometric Distributio

$$p(Z) = \left[1-P(K|H)\right]^{Z-1} \cdot P(K|H) \,, \tag{29}$$

where $P(K|H)$ = the probability of kill given a target hit, and Z = the number of target hits = 1, 2, $\cdots$. If the distribution of the number of rounds, n, required for Z hits is given by $f_{N|Z}(n|z) = P\left(n \text{ rounds} \mid \text{obtain } Z \text{ hits}\right)$, where $Z \leq n$, then the distribution of the number of rounds required to destroy the target is

$$f_N(n) = \sum_{Z=1}^{\infty} p(Z) \cdot f_{N|Z}(n|z) \,.$$

The average number of rounds, or expected number of rounds, required to destroy the target is then

$$\begin{aligned} \bar{n} &= \sum_{n=1}^{\infty} n\, f_N(n) = \sum_{n=1}^{\infty} n \left\{ \sum_{Z=1}^{\infty} p(z)\, f_{N|Z}(n|z) \right\} = \\ &\sum_{Z=1}^{\infty} p(z) \left\{ \sum_{n=1}^{\infty} n\, f_{N|Z}(n|z) \right\} = \sum_{Z=1}^{\infty} p(z)\, E\left(n|z\right) \,. \end{aligned} \tag{30}$$

S. Bonder (Ref. 1) has solved equation (30) for Markov Dependent Fire whose probability of first round hit was given by equation (27). Bonder's solution of equation (30) for Markov Dependent Fire yeilds

$$\bar{n} = \frac{1}{p \cdot P(K|H)} + \frac{\rho(1-p)}{p(1-\rho)} \,, \tag{31}$$

where p = first round probability of hit, which may be calculated from equation (25)or (26), $P(K|H)$ = probability of kill given a target hit, and ρ = the correlation between the $(n-1)^{st}$ and the n^{th} round. Bonder's solution for

Markov Dependent Fire may be applied to the case where it has been assumed that there is no correlation between rounds, i.e., $\rho = 0$, with the result that

$$\bar{n} = \frac{1}{p \cdot P(K|H)} \quad ,$$

which is equation (19).

In this chapter we used information about the artillery system from the firing tables to examine three methods for calculating single round hit probabilities for the artillery system. Methods for calculating the number of rounds to first hit, and the number of rounds to kill a target (multiple hits required) were presented for two cases. One case involved the assumption of independence between rounds and the other involved Markov Dependent Fire. With the characterizations that are developed in Chapters II and III, it is now possible for us to examine the detains of the attack of a target with both air and artillery.

IV. THE ATTACK OF A TARGET WITH THE SIMULTANEOUS USE OF AIR AND ARTILLERY

In this chapter we will examine the factors involved in attacking a target with the simultaneous use of air and artillery. The concept of the (Artillery Trajectory) Danger Air Space will be developed, as well as a method for calculating the expected total number of projectiles, bombs and artillery shells, required to destroy a target. The measure of effectiveness that will be used in this study is the expected total time required to destroy a target. Several numerical examples of the simultaneous attack of a target will be presented with varying gun-target ranges, aircraft dive angles, bomb release altitudes for several aircraft systems.

The air space in the scenario for the attack of a target with the simultaneous use of air and artillery may be separated into two mutually exclusive spaces, which we will identify as the Danger Air Space and its complement. The Danger Air Space is a portion of the air space that contains essentially all of the trajectories of the artillery that is firing on the target. An effort is made in identifying the Danger Air Space to anticipate all but the most flagrant of gunnery and ballistic errors. The use of upper 95th percentile prediction interval data, and the use of the "cookie cutter" approach in describing the artillery trajectory

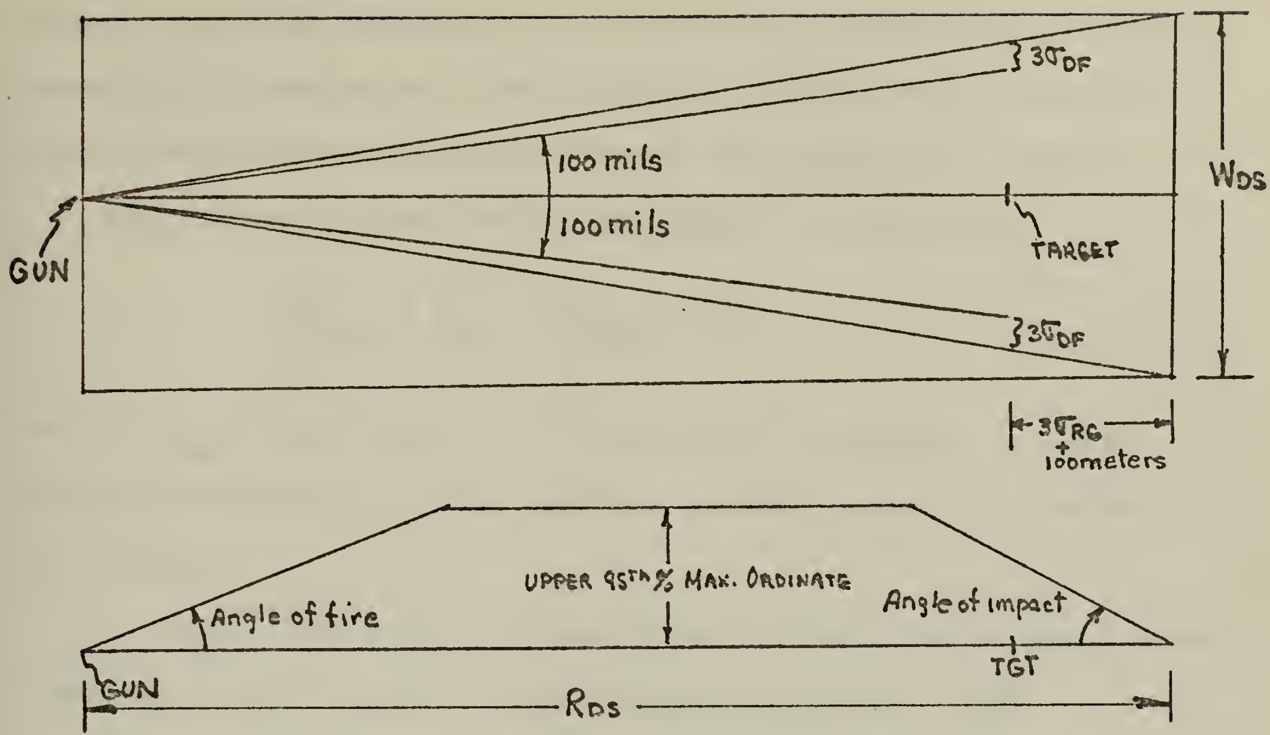


Figure 5. The Artillery Trajectory Danger Air Space

assures a Danger Air Space which will include essentially all of the artillery trajectories. Figure 5 is a sketch of the (Artillery Trajectory) Danger Air Space.

In the development of the dimensions for the Danger Air Space several assumptions were made, that if an error is going to be made on the gun and not be detected before firing it will be a 100 mil error in deflection. Another assumption is that since the last range bracket that is halved in the adjustment phase of fire is 100 meters long, that the mean point of impact will be within 50 meters of the target center. A 100 meter safety factor will be added to the Danger Air Space. The use of the upper 95th percentile prediction interval value for standard deviation of

range, standard deviation of deflection, and maximum ordinate essentially guarantees the inclusion of the worst possible case (artillery trajectory) within the Danger Air Space.

The length of the Danger Air Space is given by,

$$R_{DAS} = R_{GT} + 3\sigma_{RG} + 100 \quad , \tag{32}$$

where R_{DAS} = the length of the Danger Air Space in meters measured from the gun through the target center and beyond the target,

R_{GT} = the range between the gun and the target (often referred to as the gun-target range) in meters, and

σ_{RG} = the upper 95th percentile prediction interval standard deviation of range comensurate with the method of fire employed and such factors as change and angle of fire. The 100 meter constant is a safety factor. The width of the Danger Air Space is taken as

$$W_{DAS} = 200\ R_{DAS(K)} + 6\sigma_{DF} \quad , \tag{33}$$

where W_{DAS} = the width of the Danger Air Space centered on the target and perpendicular to the gun-target line, measured in meters,

$R_{DAS(K)}$ = the length of the Danger Air Space calculated from equation (32) measured in k-meters, and

σ_{DF} = the upper 95th percentile prediction interval standard deviation of deflection, in meters

The 200 mil coefficient converts the 100 mil deflection

safety factor (on either side of the gun-target line) into a linear measure. The angle of fire and the angle of impact are obtained from the artillery firing tables. The maximum ordinate used in the Danger Air Space is the upper 95th percentile prediction interval maximum ordinate commensurate with the method of fire employed. By using three standard deviations in range and deflection we should insure the inclusion of at least 998 rounds of every 1000 rounds fired before adding the safety factors. The 100 meter range safety factor is larger than all the standard deviations of range listed in Table 6. The 100 mil deflection safety factor results in deflection distances that are much larger than the artillery standard deviations of deflections listed in Table 6. Under the characteristics of the Normal Distribution that describes the fall of shot for artillery, and under the design of the Danger Air Space it is clear that the Artillery Trajectory Danger Air Space should include at least 9,999 rounds of every 10,000 fired.

The measure of effectiveness that will be used in this study is the expected time required to destroy the target. It will be assumed that the rate of fire of the artillery system is constant at one round per minute, which is the sustained rate of fire of the 155mm Howitzer (Ref. 5). The rate of fire (delivery) of the attack aircraft is also taken to be constant. It will be assumed that there is no correlation between the impact point of a bomb and the impact point of an artillery shell. At any time in the

attack of a target the probabilities that a projectile is a bomb or an artillery shell are calculated by,

$$P(air) = \frac{\Gamma_{air}}{\Gamma_{air} + \Gamma_{arty}} \quad , \quad \text{and} \tag{34}$$

$$P(arty) = \frac{\Gamma_{arty}}{\Gamma_{air} + \Gamma_{arty}} \quad , \tag{35}$$

where Γ_{air} = the rate of fire (delivery) of the attack aircraft in bombs per minute, and Γ_{arty} = the rate of fire of the artillery system in rounds per minute. The single round probability of hit for the combined air-artillery system is calculated by,

$$P(Hit) = P(Hit|arty) \cdot P(arty) + P(Hit|air) \cdot P(air), \tag{36}$$

where $P(Hit|arty)$ = the single round probability of hit for the artillery system which is calculated from either equation (25) or (26), $P(Hit|air)$ = the single round probability of hit for the air attack system based on calculations using equation (15), $P(air)$ and $P(arty)$ are as defined by equations (34) and (35) respectively. Under the assumption of independence between all rounds, the probability of hit on the m^{th} round is described by the Geometric Distribution,

$$P(1^{st} \text{ Hit on } m^{th} \text{ round}) = [1 - P(Hit)]^{m-1} \cdot P(Hit) \quad , \tag{37}$$

where $P(Hit)$ is given by equation (36) and $m = 1, 2, 3 \cdots$. The expected number of rounds to first hit, from equation (18) is, $\overline{m} = \frac{1}{P(Hit)}$. Since the rates of fire of the air

and artillery are determinate, then the time to first hit is given by,

$$T(1^{st}\ \text{Hit}) = \frac{1}{P(\text{Hit})\left[\Gamma_{air} + \Gamma_{arty}\right]}, \tag{38}$$

where P(Hit) is calculated from equation (36), Γ_{air} and Γ_{arty} are the rates of fire of the air and artillery systems respectively, and $T(1^{st}$ Hit) is the average time required to hit the target for the first time, in minutes.

With the multiple hits required to kill a target, the probability of kill is,

$$P\binom{\text{target}}{\text{killed}} = P\left(\text{kill}\,\middle|\,{\text{arty} \atop \text{hit}}\right)\cdot P(\text{Hit}\,|\,\text{arty})\cdot P(\text{arty}) + P\left(\text{kill}\,\middle|\,{\text{air} \atop \text{hit}}\right)\cdot P(\text{Hit}\,|\,\text{air})\cdot P(\text{air}), \tag{39}$$

where $P\left(\text{kill}\,\middle|\,{\text{arty} \atop \text{hit}}\right)$ = the probability the target is destroyed given a target hit by an artillery shell,

$P\left(\text{kill}\,\middle|\,{\text{air} \atop \text{hit}}\right)$ = the probability the target is destroyed given a target hit by a bomb,

$P(\text{hit}\,|\,\text{arty})$ = the single round probability of hit by an artillery shell, and

$P(\text{hit}\,|\,\text{air})$ = the single round probability of hit by a bomb.

Here the P(air) and P(arty) are as given by equation (34) and equation (35) respectively. Under the independence criterion the probability of kill of a target with M rounds is given by the Geometric Distribution,

$$P(\text{target killed with } M \text{ rounds}) = \left[1-P(\text{kill})\right]^{M-1} \cdot P(\text{kill}), \quad (40)$$

where P(kill) is calculated from equation (39) and $M = 1, 2, \cdots$. The expected number of rounds/bombs that are required to kill the target can be calculated from equation (18) with the result that $\overline{M} = 1/P(\text{kill})$. The conversion of the number of rounds/bombs required to kill the target to the time required to kill the target is

$$T(\text{kill target}) = \frac{1}{P(\text{kill})\,(\Gamma_{air} + \Gamma_{arty})}, \quad (41)$$

where P(kill) is calculated from equation (39), Γ_{air} and Γ_{arty} are the rates of fire of the air and artillery systems respecitvely, and T(kill target) is the time in minutes required to kill the target.

At this point in the analysis we have developed the relationships and concepts that are needed to investigate both the dangers and the advantages of attacking a target with simultaneous air delivery and artillery. This will be done with a series of examples involving various combinations of mode of attack by the attack aircraft and the artillery guns.

Example 1. The basis for example 1 is a gun-target range of 4000 meters, a 10 mil aircraft system, and a 26 mil aircraft system. Table 6 is an abstract of data from Table 5 that will be needed in the development of this example.

TABLE 6

Possible Firing Data for a Gun-Target Range of 4,000 Meters

Charge	PER (m.)	PED (m.)	95% Max.Ord. (m.)	Angle Fire (deg.)	Angle Impact (deg.)	RG (95%) (m.)	DF (95%) (m.)
1	20	2	950	37.1	39.8		
2	17	2	640	24.5	27.5	25.2 (37.8)	3.0 (4.4)
3	14	2	460	17.8	19.3	20.8 (31.1)	3.0 (4.4)
4	11	2	340	13.1	14.3	16.3 (24.5)	4.4 (6.7)
5	9	2	270	10.1	13.5		
6	18	2	200	7.0	8.9		

The charges that will be used in this example are green 2, green 3, and green 4, which are the charges that are most likely to be chosen by the firing battery Fire Direction Officer. The dimensions of the Danger Air Space as calculated from equations (32) and (33), and as abstracted from Table 6 for charges two, three and four are listed in Table 7. The Table 7 values for R_{DAS} and W_{DAS} have been rounded up. Comparison of the maximum ordinate data in Table 7 with the minimum altitude data contained in Table 4 reveals that aircraft dropping their bombs from 5000 feet or higher altitudes from 45^{o} or 30^{o} dives with a minimum 3-G pull-up will not penetrate the Danger Air Space regardless of their azimuth of approach.

TABLE 7

The Dimensions of the Danger Air Space

CHARGE	R_{DAS} METERS	W_{DAS} METERS	FEET	ANGLE OF FIRE	ANGLE OF IMPACT	UPPER 95% MAX. ORD. METERS	FEET
2	4220	880	2887	24.5^{o}	27.5^{o}	640	2099
3	4200	870	2854	17.8^{o}	19.3^{o}	460	1509
4	4180	880	2887	13.1^{o}	14.3^{o}	340	1115

At this point we wish to assess the feasibility of attacking the target with aircraft that are releasing their bombs at 3000 feet during a 45^{o} dive. For the aircraft to release their bombs below 5000 feet it becomes necessary for the Forward Air Controller to restrict the azimuth of the attack aircraft approach. In determining the azimuths that are feasible we will base out calculations on a bomb miss distance of three standard deviations of CEP (recall that CEP = $1.1774\sigma_{CEP}$) from the target in the direction of maximum probability of interaction with the Danger Air Space. This is commensurate with worst case planning.

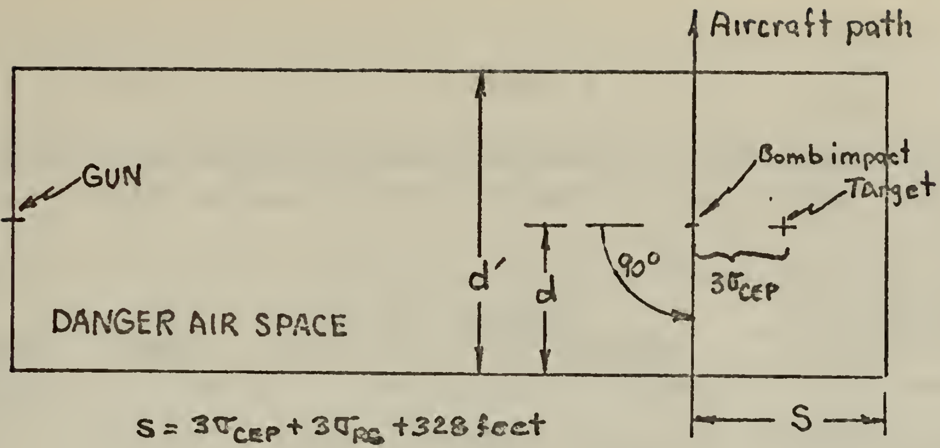


Figure 6. Aircraft 90° to Gun-Target Line, Level Flight, Danger Air Space Interaction

TABLE 8

Results of Aircraft/Danger Air Space Interaction for 90° Azimuth
(All values in feet except charge and ∡ impact)

CHARGE	∡ IMPACT	10 MIL SYSTEM d	d'	S	ALT@S	26 MIL SYSTEM d	d'	S	ALT@S
2	27.5°	1450	2900	845	439.9	1450	2900	1041	542.1
3	19.3°	1450	2900	780	273.3	1450	2900	975	341.6
4	14.3°	1450	2900	715	182.2	1450	2900	910	232.0

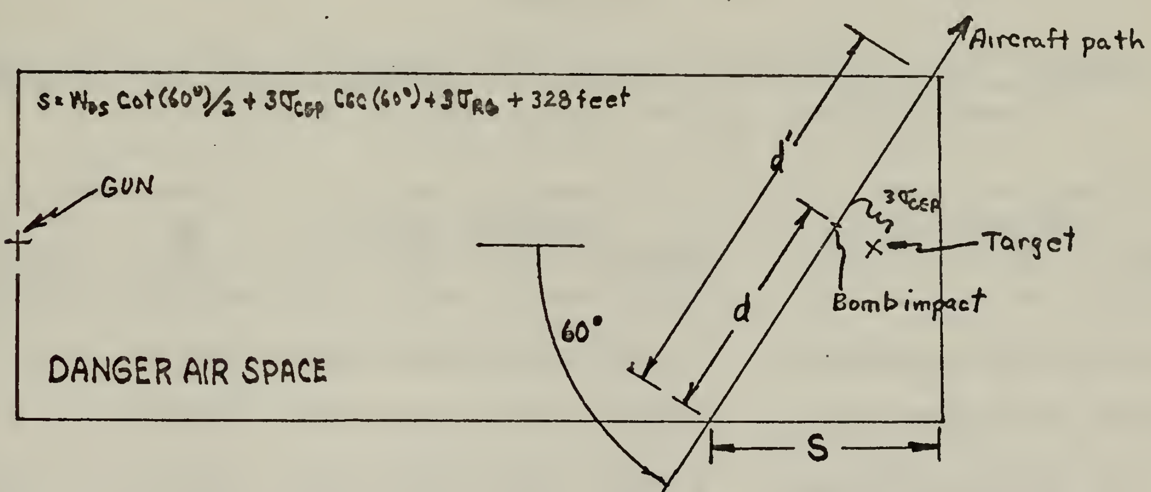
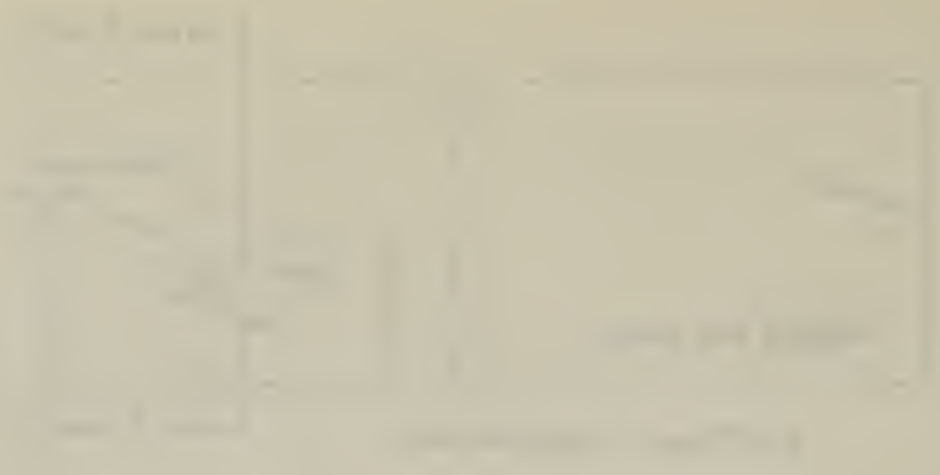


Figure 7. Aircraft 60° to Gun-Target Line, Level Flight, Danger Air Space Interaction



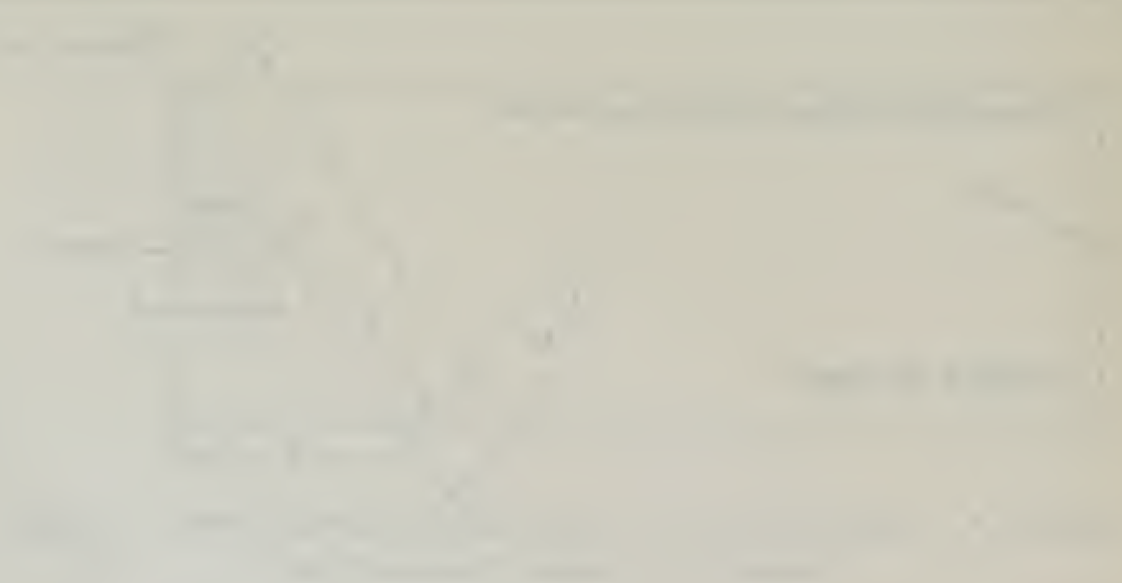


TABLE 9

Results of Aircraft/Danger Air Space Interaction, 60° Azimuth
(All values in feet except charge and ∡ impact)

CHARGE	∡ IMPACT	10 MIL SYSTEM d	d'	S	ALT@S	26 MIL SYSTEM d	d'	S	ALT@S
2	27.5°	1757	3350	1702	886.0	1872	3350	1931	1005.4
3	19.3°	1757	3272	1636	572.9	1872	3350	1865	653.2
4	14.3°	1757	3142	1571	400.2	1872	3350	1800	458.9

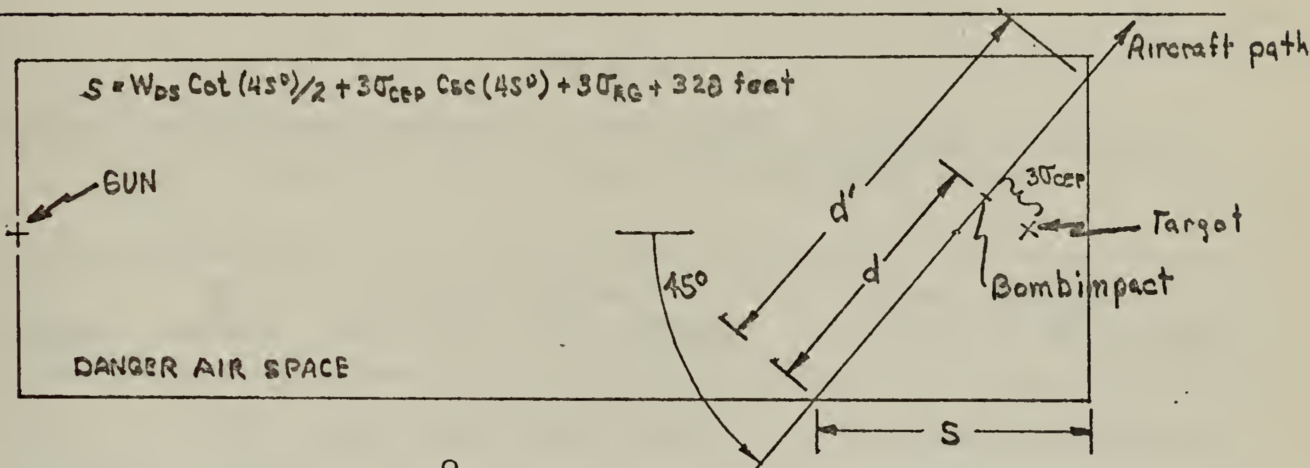
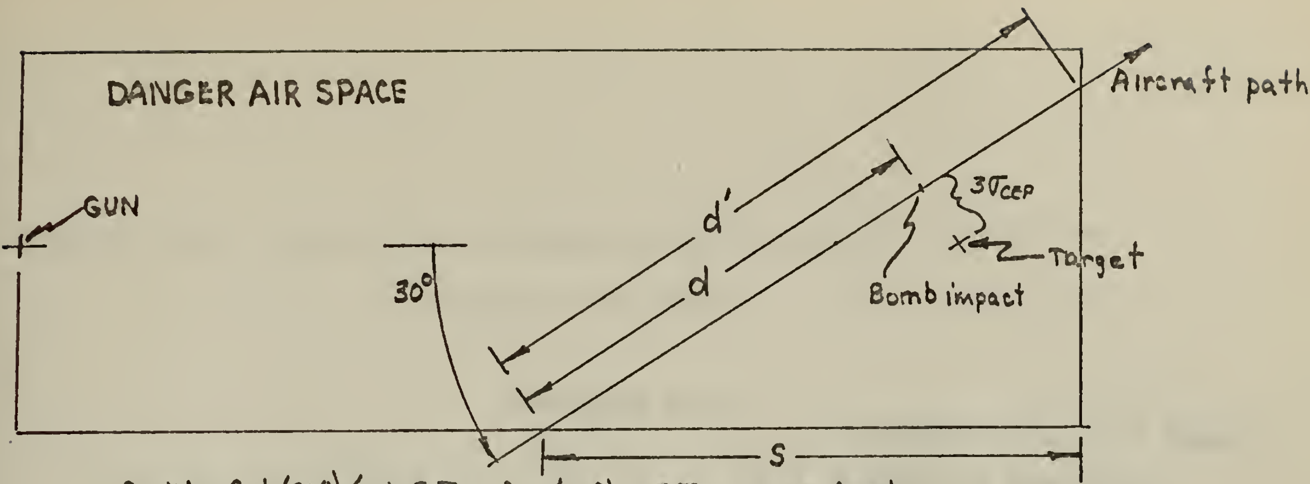


Figure 8. Aircraft 45° to Gun-Target Line, Level Flight, Danger Air Space Interaction

TABLE 10

Results of Aircraft/Danger Air Space Interaction, 45° Azimuth
(All values in feet except charge and ∡ impact)

CHARGE	∡ IMPACT	10 MIL SYSTEM d	d'	S	ALT@S	26 MIL SYSTEM d	d'	S	ALT@S
2	27.5°	2193	3323	2352	1224.3	2392	3721	2633	1370.5
3	19.3°	2193	3240	2286	800.5	2392	3492	2567	898.9
4	14.3°	2193	3142	2221	566.1	2392	3398	2502	637.7



$$S = W_{DS}\,\mathrm{Cot}(30°)/2 + 3\sigma_{CEP}\,\mathrm{Csc}(30°) + 3\sigma_{RG} + 328 \text{ feet}$$

Figure 9. Aircraft 30° to Gun-Target Line, Level Flight Danger Air Space Interaction

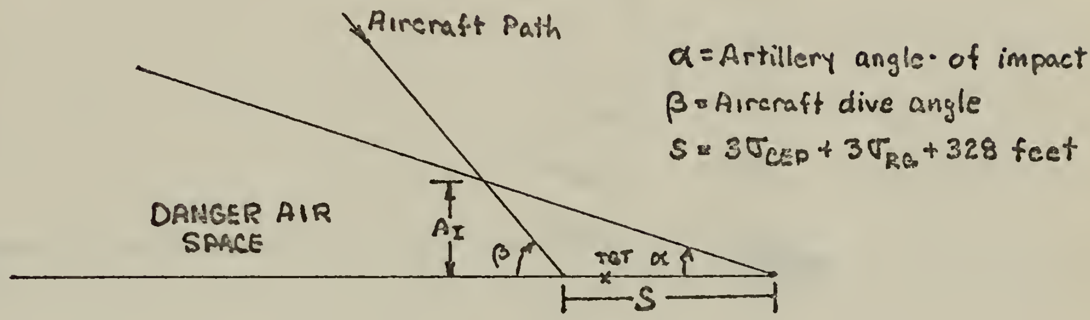
TABLE 11

Results of Aircraft/Danger Air Space Interaction, 30° Azimuth
(All values in feet except charge and ∡ impact)

	∡	10 MIL SYSTEM				26 MIL SYSTEM			
CHARGE	IMPACT	d	d'	S	ALT@S	d	d'	S	ALT@S
2	27.5°	3147	4041	3497	1820.4	3491	4490	3894	2027.1
3	19.3°	3147	3962	3431	1201.5	3491	4420	3828	1340.6
4	14.3°	3147	3897	3366	858.0	3491	4348	3763	959.2

For an aircraft traveling along the gun-target line in level flight the maximum ordinate encountered is 2100 feet for charge 2, 1510 feet for charge 3, and 1120 feet for charge 4. Figure 10 pictures the worst case interaction for an aircraft moving along the gun-target line and diving at 45°.

Figure 10. Interaction Between Diving Aircraft and the Danger Air Space



The altitude at which the aircraft intrudes into the Danger Air Space, A_I, is calculated from,

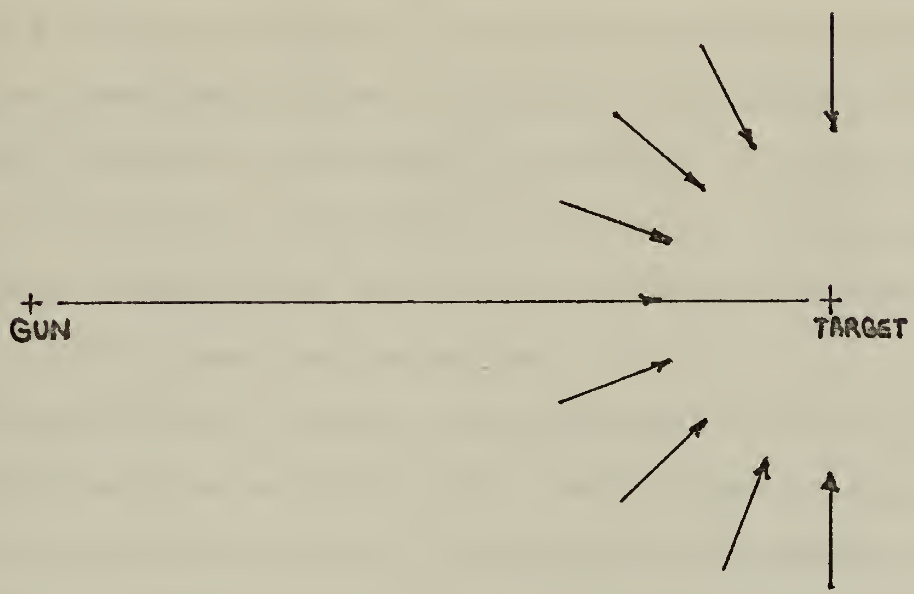
$$A_I = \frac{S \sin\alpha \sin\beta}{\sin(\beta - \alpha)} ,$$

where α = the angle of impact of the artillery shell,

β = the dive angle of the aircraft, and

S = the distance from the edge of the Danger Air Space through the target as pictured in Figure 10. For the data that characterizes this example, A_I is equal to 917.5 feet for charge 2, 420.4 feet for charge 3, and 244.6 feet for charge 4. By superimposing the standoff range data in Table 3 over the aircraft path relative to the impact point in Figures 6, 7, 8, and 9, and then by carefully analyzing the implications of minimum altitude and the position at which the minimum altitude occurs (as per Table 4), it is

Figure 11. Safe Azimuth of Attack for Bomb Dropped Below 5000' and Above 3000' from 30° or 40° Dives with 3-G, 4-G, or 5-G pull up.



clear that any aircraft approach azimuth at $\pm 90^{\circ}$ to the gun-target line will be safe, i.e., it will not penetrate the Danger Air Space for bomb delivered at 3000 feet or higher altitudes from 30° or 45° dive angles. Pull up at 3-G's or higher are required, with 4-G or 5-G pull up preferred, to allow an even greater margin of safety. Figure 11 diagrams the safe approach azimuths for the aircraft attacking the target.

It should be noted that the maximum ordinate of the Danger Air Spaces for this scenario allows the attack of the target from any azimuth for aircraft in level flight under TPQ-27 Radar control at 2600 feet or higher altitude without any danger of penetrating the Danger Air Space.

Now that we have determined the modes of attack that are feasible for the attack aircraft, we may assess the advantages of attacking the target with the air-artillery system. For this analysis we shall consider, 1) a circular target with a 20 foot diameter, 2) a rate of fire for the artillery of one round per minute, 3) a rate of delivery for the attack aircraft of one bomb per minute, 4) a P(kill|artillery hit) of 0.25, 5) a P(kill|air hit) of 0.75, 6) a gun-target range of 4000 meters, and 7) the 155mm Howitzer fires charges 2, 3, and 4 green bag propellant.

The artillery single shot probability of a hit is calculated from equation (26). The attack aircraft single shot probability of hit is calculated from equation (15). Table 12 contains the results of the calculations that are required to assess the advantages of attacking the target with air and artillery. Listed in Table 12 are the expected times to destroy the target for the artillery system, the 10 mil, and 26 mil aircraft systems, and all combinations of the air-artillery system. Analysis of the percentage of time saved for the combined air-artillery system relative to the air system shows the advantage that is gained through the use of the combined system, as a direct result of effectively doubling the rate of fire on the target. A 30.1% reduction in the time to destroy the target from the air point of view, implies that less air sorties are needed. Additionally, the smaller the number of sorties required to destroy the target, the smaller the probability of losing an aircraft during the attack on the target.

TABLE 12

Summary of Results for Example 1

10 MIL A/C SYSTEM σ_{CEP}	P(HIT)	ALT.	DIVE ANGLE	P(KILL|HIT)	26MIL A/C SYSTEM σ_{CEP}	P(HIT)
73.0	0.0093	3000	30°	0.75	173.6	0.0017
47.6	0.0218	3000	45°	0.75	113.8	0.0039
105.2	0.0045	5000	30°	0.75	250.6	0.0008
74.4	0.0090	5000	45°	0.75	177.5	0.0016

155mm Howitzer System

CHARGE	σ_{RG}	σ_{DF}	P(HIT)	UPPER 95% σ_{RG}	UPPER 95% σ_{DF}	P(HIT)	P(KILL|HIT)
2	25.2	3.0	0.0615	37.8	4.4	0.0279	0.25
3	20.8	3.0	0.0745	31.1	4.4	0.0340	0.25
4	16.3	4.4	0.0648	24.5	6.7	0.0283	0.25

EXPECTED TIME TO KILL THE TARGET, IN MINUTES

		10 MIL A/C SYSTEM				26 MIL A/C SYSTEM				% TIME SAVED RELATIVE TO ARTY.	
		30° DIVE 3000'	45° DIVE 3000'	30° DIVE 5000'	45° DIVE 5000'	30° DIVE 3000'	45° DIVE 3000'	30° DIVE 5000'	45° DIVE 5000'		
	AIR ONLY	143.4	61.2	296.3	148.1	748.3	341.9	1666.7	833.3	MIN.	MAX.
ARTY. CHARGE	ARTY. ONLY										
2	65.0	44.7	31.5	53.3	45.2	60.0	54.6	62.6	60.3	3.6	51.5
UPPER 95% 2	143.4	71.6	42.8	96.5	72.7	121.1	101.0	131.9	122.2	8.0	70.2
3	53.7	39.1	28.6	45.5	39.4	50.3	46.4	52.0	50.5	3.2	46.7
UPPER 95% 3	117.6	64.6	40.2	84.2	65.5	102.2	87.6	109.9	103.1	6.5	65.8
4	61.7	43.1	30.7	51.1	43.6	57.2	52.3	59.5	57.5	3.6	50.2
UPPER 95% 4	141.3	71.1	42.7	95.6	72.3	119.6	100.0	130.2	120.8	7.9	69.8

% TIME SAVED RELATIVE TO AIR

MIN	50.1	30.1	67.4	50.9	83.8	70.5	92.1	85.3
MAX	72.7	53.3	84.6	73.4	992.3	86.4	96.9	93.9

NUMBER OF AIRCRAFT PASSES SAVED

MIN	71.8	18.4	199.8	75.4	627.2	240.9	1534.8	711.1
MAX	104.3	32.6	250.8	108.7	698.0	295.5	1614.7	782.8

Example 2. The basis for this example is a gun-target range of 8,000 meters, a 10 mil aircraft system, and a 26 mil aircraft system. Table 13 contains possible artillery firing data for a gun-target range of 8,000 meters.

TABLE 13

155mm Howitzer Firing Data for 8,000 Meter Gun-Target Range

Charge	PER (m.)	PED (m.)	95% Max.Ord. (m.)	Angle Fire (deg.)	Angle Impact (deg.)	RG (95%) (m.)	DF (95%) (m.)
5G	19	4	1200	25.1	29.8	28.2 (42.3)	5.9 (8.9)
6W	27	4	910	18.2	23.7	40.0 (60.0)	5.9 (8.9)
7W	24	3	660	12.6	18.7	35.6 (53.4)	4.4 (6.7)
8	23	2	430	8.0	12.8	34.1 (51.2)	3.0 (4.4)

For the example charges 6 white and 7 white will be used. The dimensions of the Danger Air Space, as calculated from equations (32) and (33), are listed in Table 14.

TABLE 14

Danger Air Space Dimensions for a Gun-Target Range of 8,000 M.

CHARGE	R_{DAS} METERS	W_{DAS} METERS	W_{DAS} FEET	ANGLE OF FIRE	ANGLE OF IMPACT	UPPER 95% MAX. ORD. METERS	UPPER 95% MAX. ORD. FEET
6	8280	1709	5606	18.2	23.7	910	2985
7	8260	1692	5550	12.6	18.7	660	2165

Based on the data contained in Table 4 and the maximum ordinate data for charges 6 and 7 , unrestricted attack of the target by aircraft is possible for 30^{o} dives at altitudes of release of 5,000 feet or higher and for 45^{o} dives at release altitudes of 6,000 feet or higher. Table 15 contains the interaction data between the Danger Air Space and the attack aircraft for aircraft approaching from 90^{o}, 60^{o}, 45^{o}, and 30^{o} to the gun-target line. The data in Table 15 is based on 3,000 foot release CEP for a 10 mil and a 26 mil aircraft system.

By superimposing the standoff range data from Table 3 relative to the bomb impact point, in conjunction with the minimum altitude and position of minimum altitude data from Table 4 for the various approach azimuths in Table 15, it is apparent that all modes of attack, 30^{o} or 45^{o} dive with releases at 3,000 feet or higher, at $\pm 90^{o}$ or less to the gun-target line, are safe. The only exception to this result is that all 45^{o} dive, 3,000 foot release attacks made between 60^{o} and 90^{o} require the additional restriction of a 4-G or 5-G pull up. The approach azimuth diagram for Example 1, Figure 11, is applicable to this example with the previously mentioned additional requirement on the 45^{o} at 3,000 feet between 60^{o} and 90^{o}.

Having determined the safe modes of attack for the attack aircraft, we are ready to assess the advantages of attacking the target with the combined air-artillery system. For the purpose of this analysis we will consider, 1) a circular

TABLE 15

Danger Air Space/Attack Aircraft Level Flight Interaction Data

Basis: 10 mil A/C σ_{CEP}=47.6 ft., 26 mil A/C σ_{CEP}=113.8 ft.

(All values in feet except charge and angle of impact)

CHARGE	ANGLE OF IMPACT	10 MIL SYSTEM d	10 MIL SYSTEM d'	10 MIL SYSTEM S	10 MIL SYSTEM ALT@S	26 MIL SYSTEM d	26 MIL SYSTEM d'	26 MIL SYSTEM S	26 MIL SYSTEM ALT@S
a) Aircraft Approach at 90° to Gun-Target Line, (Figure 6)									
6	23.7	2800	5600	1061	465.7	2800	5600	1260	553.1
7	18.7	2800	5600	996	337.1	2800	5600	1195	404.5
b) Aircraft Approach at 60° to Gun-Target Line, (Figure 7)									
6	23.7	3316	5400	2700	1185.2	3430	5858	2929	1285.7
7	18.7	3316	5270	2635	891.9	3430	5728	2864	969.4
c) Aircraft Approach at 45° to Gun-Target Line, (Figure 8)									
6	23.7	4103	5544	3920	1720.8	4301	5941	4201	1844.1
7	18.7	4103	5453	3856	1305.2	4301	5849	4136	1400.0
d) Aircraft Approach at 30° to Gun-Target Line, (Figure 9)									
6	23.7	5816	6990	6054	2657.5	6191	7449	6451	2831.8
7	18.7	5816	6915	5989	2027.2	6191	7374	6386	2161.5

e) Aircraft Approach at 0° to Gun-Target Line in a 45° Dive, (Figure 10)

CHARGE	10 MIL SYSTEM S	10 MIL SYSTEM A_I	26 MIL SYSTEM S	26 MIL SYSTEM A_I
6	1061.2	830.3	1259.8	985.7
7	996.3	509.8	1194.9	611.4

target 30 feet in diameter, 2) a rate of fire for the 155mm Howitzer of one round per minute, 3) a rate of delivery for the attack aircraft of one bomb per minute, 4) a probability of kill given an artillery hit of 0.30, 5) a probability of kill given an air hit of 0.80, 6) a gun-target range of 8,000 meters, and 7) the Howitzer using charges 6 and 7 white bag propellent.

The single shot probability of hit for both the air and artillery system is calculated from equation (26). The expected time to kill the target is determined from equation (41) for the air attack, artillery attack, and the air-artillery attack system. Table 16 is a summary of hit probabilities, and expected time to kill the target data for the 155mm Howitzer artillery system, a 10 mil attack aircraft system, a 26 mil attack aircraft system, and for all combinations of the air-artillery attack system.

As was the case in Example 1, we again realize a significant saving in the expected time to destroy the target relative to the air system, due to the effective doubling of the rate of fire on the target. This implies that fewer sorties are required to destroy the target, and hence less exposure of the attack aircraft to the hostile environment of the battle area. In the next chapter we will discuss some of the advantages and disadvantages of the combined air-artillery attack system.

TABLE 16

Summary of Results for Example 2

10 MIL A/C SYSTEM		ALT.	DIVE ANGLE	P(KILL|HIT)	26 MIL A/C SYSTEM	
σ_{CEP}	P(HIT)				σ_{CEP}	P(HIT)
73.0	0.0211	3000	30^{o}	0.80	173.6	0.00371
47.6	0.0496	3000	45^{o}	0.80	113.8	0.00869
105.2	0.0101	5000	30^{o}	0.80	250.6	0.00178
74.4	0.0203	5000	45^{o}	0.80	177.5	0.00356

155mm Howitzer System

CHARGE	σ_{RG}	σ_{DF}	P(HIT)	UPPER 95% σ_{RG}	UPPER 95% σ_{DF}	P(HIT)	P(KILL|HIT)
6	40.0	5.9	0.0443	60.0	8.9	0.0196	0.30
7	35.6	4.4	0.0668	53.4	6.7	0.0292	0.30

EXPECTED TIME TO KILL THE TARGET, IN MINUTES

		10 MIL A/C SYSTEM				26 MIL A/C SYSTEM				% TIME SAVED RELATIVE TO ARTY.	
		30° DIVE 3000'	45° DIVE 3000'	30° DIVE 5000'	45° DIVE 5000'	30° DIVE 3000'	45° DIVE 3000'	30° DIVE 5000'	45° DIVE 5000'		
	AIR ONLY	59.2	25.2	123.8	61.6	336.9	143.8	702.2	351.1	MIN.	MAX.
ARTY. CHARGE	ARTY. ONLY										
6	75.2	33.1	18.2	46.8	33.9	61.5	49.4	68.0	62.0	9.6	75.8
UPPER 95% 6	170.1	43.9	21.9	71.6	45.2	113.0	77.9	136.9	114.6	19.5	87.1
7	49.9	27.1	16.7	35.6	27.2	43.5	37.0	46.6	43.7	6.6	66.5
UPPER 95% 7	114.2	39.0	20.6	59.4	40.0	85.3	63.9	98.2	86.1	14.0	82.0

% TIME SAVED RELATIVE TO AIR

	MIN	25.8	13.1	42.2	26.6	66.5	45.8	80.5	67.4
	MAX	54.2	33.7	71.2	55.2	87.1	74.3	93.4	87.6

NUMBER OF AIRCRAFT PASSES SAVED

	MAX	32.1	8.5	88.2	34.4	293.4	106.8	656.6	307.4
	MIN	15.3	3.3	52.2	16.4	223.9	65.9	565.3	236.5

V. DISCUSSION OF RESULTS AND RECOMMENDATIONS FOR FURTHER STUDY

In this chapter we will consider the restrictions that must be placed on the artillery system in the combined attack, and then based upon the artillery system restrictions, we will develop the air system restrictions. We will then consider the advantages and disadvantages of using the simultaneous air-artillery attack system. The report will be concluded with some recommendations for further study.

One restriction that must be placed on the artillery system is that no V-T fuzed artillery shells may be used in the attack of any target in the vicinity of the area where the combined air-artillery attack system is being used. These fuzes were originally designed for use with anti-aircraft artillery projectiles, and therefore are extremely sensitive to any aircraft or other material that might be in the air. The exclusion of V-T fuzes greatly decreases the possibility that a distant aircraft might be damaged by "friendly" artillery shells. A second restriction on the artillery system is that the angle of impact of the artillery shells must be less than or equal to 30^{o} (533 mils). Table 17 lists the maximum gun-target range, the highest 95^{th} percentile maximum ordinate, the largest 95^{th} percentile standard deviation of range, and a representative value of the terminal velocity of the artillery projectile for

charges 1 green bag to 5 green bag, 6 white bag, 7 white bag and charge 8. The data that is contained in Table 17 will be used to develop the attack aircraft restrictions.

TABLE 17

Artillery Restrictions

Charge	Maximum Gun-Target Range	Largest Upper 95% σ_{RG}	Highest Upper 95% Max. Ord.	Rep. Terminal Velocity	Rel. Veloc. to an A/C Moving at 450 Knots
	(meters)	(meters)	(feet)	(knots)	(knots)
1G	3,500	37.8	2,035	379	-71
2G	4,200	40.6	2,395	420	-30
3G	5,200	43.6	2,707	464	14
4G	6,700	44.0	3,488	517	67
5G	8,000	42.3	3,937	554	104
6W	9,200	71.0	4,233	580	130
7W	10,500	62.3	4,364	622	172
8	12,500	75.6	4,987	640	190

In keeping with the tenet of basing conclusions on worst case planning, the Danger Air Space width will <u>not</u> be given any limit. This will allow other artillery units to attack any other target that is short of a line through the air-artillery target, and perpendicular to the Danger Air Space gun-target line, as long as the angle of impact of the other artillery units' fire is less than 30° (533 mils). Table 18 lists the data that results from interacting a 26 mil

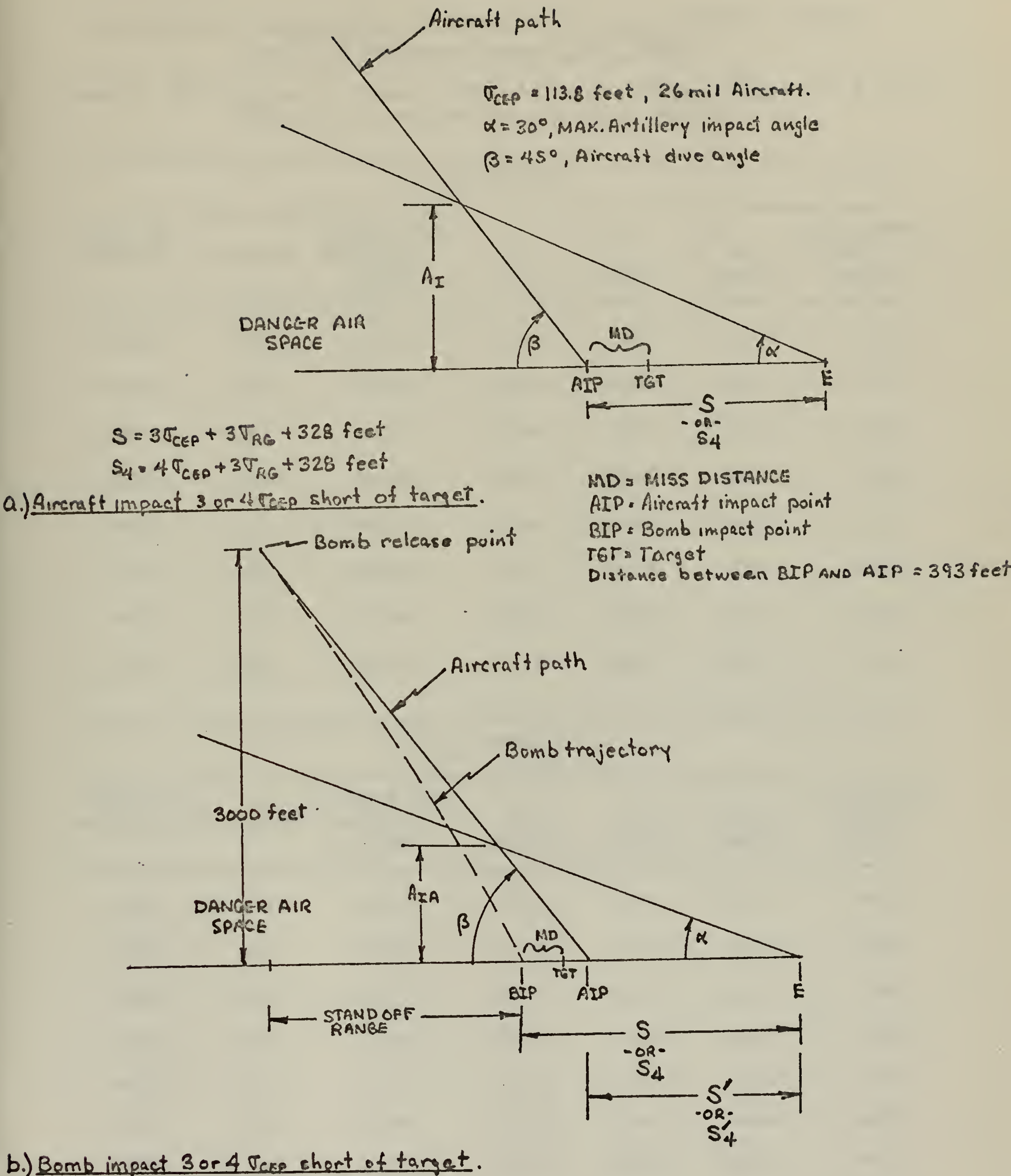


Figure 12. Attack Aircraft in 45° Dive Interacting with 30° Angle of Impact Danger Air Space.

TABLE 18

Results of Interaction Between a 26 mil Aircraft System and the Maximum Danger Air Space

Basis: 30^{o} Angle of Impact, 45^{o} Aircraft Dive, σ_{CEP}=113.8 Feet
(Notation defined on previous page)

$3\sigma_{CEP}$ RESULTS

TPQ 27 MIN.ALT.	CHARGE	UPPER 95% MAX.ORD. (FEET)	S (FEET)	A_I (FEET)	A_{IA} (FEET)	ALT.OVER BIP (FEET)
2500	1G	2035	1042	1423	887	602
2900	2G	2395	1069	1460	928	617
3200	3G	2707	1095	1491	959	632
4000	4G	3488	1103	1507	970	637
4400	5G	3937	1086	1483	947	627
4700	6W	4233	1368	1869	1332	790
4900	7W	4364	1283	1753	1216	741
5500	8	4987	1414	1932	1395	816

$4\sigma_{CEP}$ RESULTS

TPQ 27 MIN.ALT.	CHARGE	UPPER 95% MAX.ORD. (FEET)	S_4 (FEET)	A_{I4} (FEET)	A_{IA4} (FEET)	ALT.OVER BIP_4 (FEET)
2500	1G	2035	1156	1579	1042	667
2900	2G	2395	1183	1616	1079	683
3200	3G	2707	1209	1651	1115	698
4000	4G	3488	1217	1662	1126	703
4400	5G	3937	1200	1639	1102	693
4700	6W	4233	1482	2024	1488	856
4900	7W	4364	1397	1908	1371	807
5500	8	4987	1528	2087	1550	882

aircraft system with the Danger Air Space that is given by the data contained in Table 17.

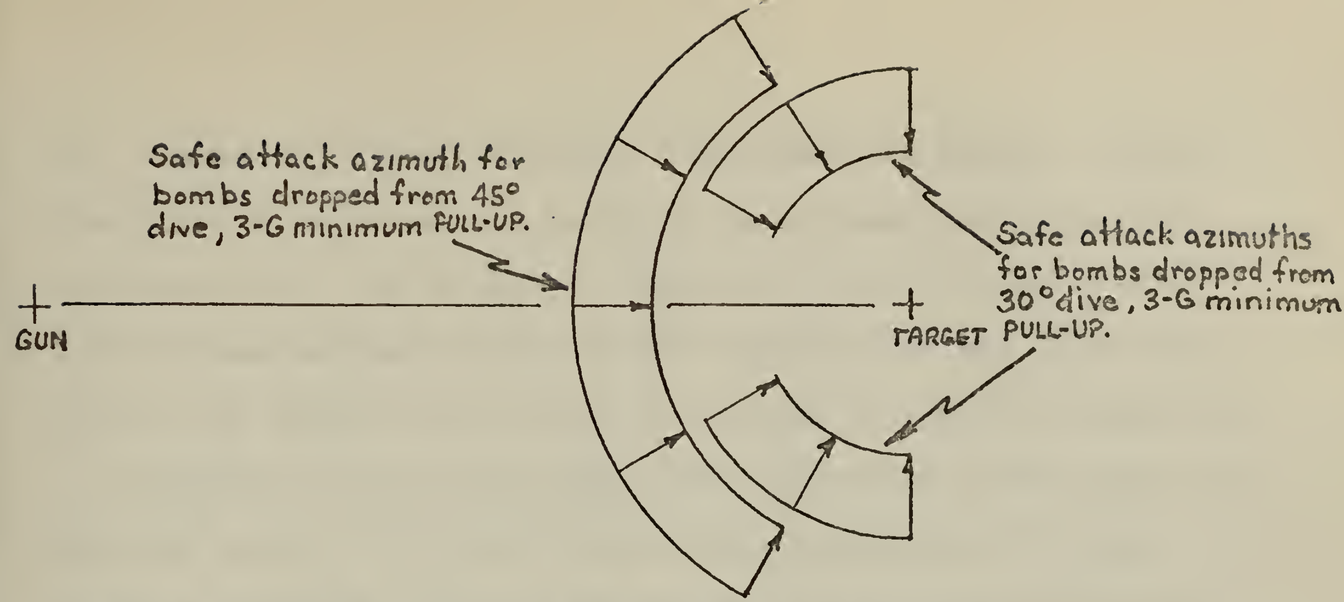
By carefully interacting the data contained in Table 18 with the minimum altitude and position of the aircraft at minimum altitude data contained in Table 4, we can determine the attack azimuths that are safe for the attack aircraft. Figure 13 is a graphical representation of the safe azimuths for the attack aircraft relative to the gun-target line. Table 19 contains a summary of safe azimuths for the attack aircraft. The upper 95% maximum ordinate data with a 500 foot safety factor represents the level flight TPQ-27 radar air drop without any azimuth restrictions (Table 18). The safe azimuths given in Table 19 are based on a 10° to 15° azimuth miss safety factor.

TABLE 19

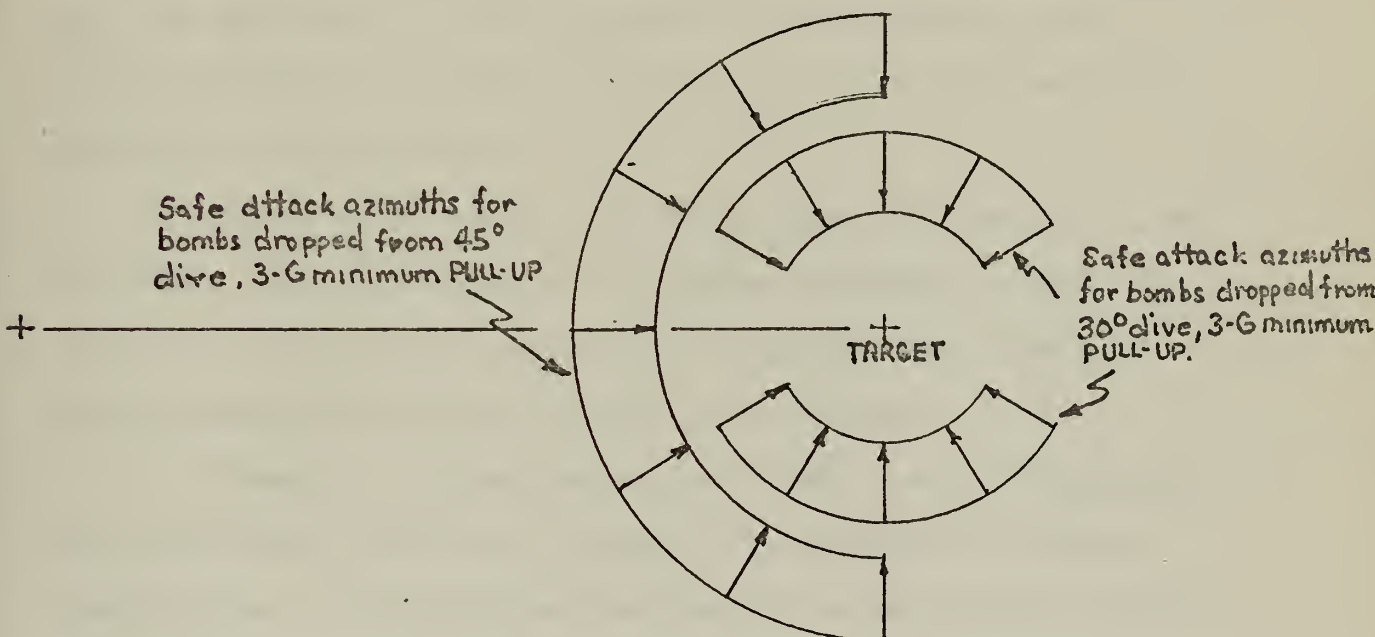
Safe Azimuths for Bomb Releases at 3000, 4000 and 5000 Feet

Basis: 3G minimum pull-up, max. bomb miss distance of $4\sigma_{CEP}$.

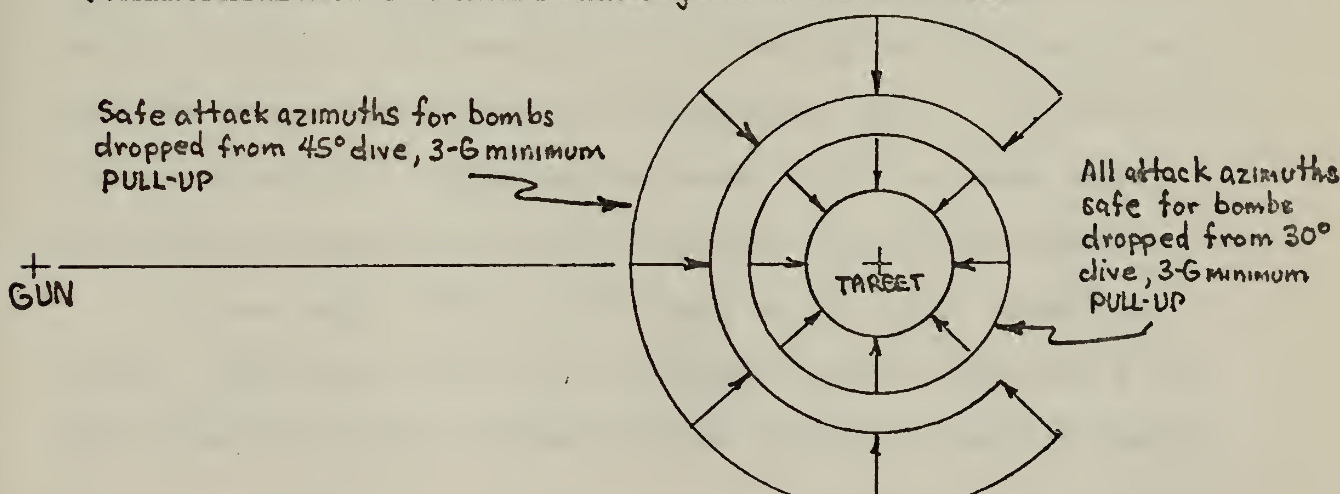
RELEASE ALTITUDE (FEET)	SAFE AZIMUTHS FOR BOMBS DROPPED FROM 45° DIVE (DEGREES RELATIVE TO GUN-TARGET LINE)	SAFE AZIMUTHS FOR BOMBS DROPPED FROM 30° DIVE (DEGREES RELATIVE TO GUN-TARGET LINE)
3000	$\pm 60^{\circ}$	-30° to -90°, and $+30^{\circ}$ to $+90^{\circ}$
4000	$\pm 90^{\circ}$	-30° to -150°, and $+30^{\circ}$ to $+150^{\circ}$
5000	$\pm 135^{\circ}$	360°



a) Safe azimuths for aircraft releasing bombs at 3000 feet



b) Safe azimuths for aircraft releasing bombs at 4,000 feet.



c) Safe azimuths for aircraft releasing bombs at 5,000 feet

Figure 13. Safe Azimuths for Bombs Released from 3000 to 5000 ft.

For example , if an aircraft released its bombs at 3000 feet from a 45° dive, it could do so without any danger of intruding into the Danger Air Space as long as its attack azimuth was between $+75^{\circ}$ and -75° relative to the gun-target line, even though the azimuth is limited to $\pm 60^{\circ}$ by Table 19.

Now that we have developed the restrictions that must be imposed on the air attack and artillery systems, we are ready to consider the advantages and disadvantages that are involved in attacking a target with the simultaneous use of air and artillery. First we shall examine some of the disadvantages associated with the use of the combined air-artillery attack system.

1. There is a limitation on the type of fuzes that may be used. This is really not a severe drawback in that VT fuzes are not normally used in a situation where we are interested in destroying a hard point target.

2. There are range limitations that must be imposed on the artillery system as a result of the angle of impact restriction. In precision destruction missions the choice of the charge that is to be used is based upon the location of the target, problems with intervening terrain features, and the range and deflection dispersion data. The angle of impact limitation includes the best possible range and deflection dispersion characteristics for all charges.

3. Low angle fire is the only method that should be used. High angle fire (firing angle greater than 45°) is not normally used in attempting to destroy a hard target in

that high angle fire involves very large range and deflection dispersion.

4. The attack aircraft are limited in the azimuths that they may use in low altitude attacks against the target.

5. Another factor is the aircraft pilots' feelings of apprehension of artillery shells in proximity to their aircraft. The pilots' apprehension is probably due to a lack of understanding. Proper education might eliminate or effectively counter this problem.

6. Implementation of this method of attack will require close cooperation between the air, artillery, and infantry. The advent of the universal forward observer, and the integrated Fire Support Coordination/Direct Air Support function will do much to smooth out the liaison problems that are implied in the use of this method of attack.

7. The artillery must be in, or ready to begin, the Fire-for-Effect phase of fire before the air strikes begin. This is due to the fact that during the Adjustment phase of fire, it is not unusual that initial bold range and deflection changes of 400 to 600 meters might be required in establishing a fall of shot bracket over the target. This requirement of being in, or ready for, Fire-for-Effect is not a serious problem in that the artillery should be well into the Fire-for-Effect phase before the strike aircraft arrive on station and are ready to begin their strikes.

Having considered some disadvantages of the combined air-artillery attack system, we are ready to examine some

advantages that are realized as a result of employing the combined air-artillery attack system over the old method of firing artillery, interrupting the artillery fire, bringing in the air strikes, delaying some more, then resuming the artillery fire. The advantages of the combined air-artillery attack include:

1. Once the attack on the target begins, it will not be interrupted until the target is destroyed. There is no need to interrupt the artillery fire to allow the air strikes to begin. If the target is not destroyed after the attack aircraft have expended their last bomb, the artillery can continue firing until the target is destroyed.

2. Target acquisition by the air strike force is greatly enhanced. The artillery can provide target replot data to the attack aircraft, which may then be used as an input into the aircraft avionics system. The artillery bursts may also be used by the Forward Air Controller as a reference point for the target location and identification.

3. As a direct result of the reduction in the expected time to destroy the target, we may infer that fewer aircraft attack passes will be required on the average to destroy the target. This means that there is a reduction in the number of sorties that are required to destroy the target over what would be required if only air were used to destroy the target. Since fewer sorties are required, it is apparent that the aircraft will have a shorter total exposure time, over the total exposure for the pure air attack case, and

therefore on a target destroyed basis, the expected cost in lost aircraft will be lower. If the target is impeding the advance of an infantry force, then the quicker we can eliminate the target, the quicker the infantry force can continue to move, and therefore the shorter the infantry's exposure time to enemy fire.

4. Other artillery pieces may attack any other target that falls within the envelope of the Danger Air Space. This allows the immediate attack of any anti-aircraft system that might become apparent during the air operations in the battle area. The more that we are able to reduce the enemy's capability to bring fire against our attack aircraft, the greater will be our savings in terms of the cost of the target destruction.

5. Time-on-target techniques may be used with the TPQ-27 radar controlling the air dropped ordnance in coordination with the artillery fire control system. This type of time-on-target technique involves a greater shock effect, larger target area coverage, and can result in greater damage to the enemy force without any loss of surprise as compared to the effects of the normal pure artillery time-on-target fire.

Having examined some of the advantages and disadvantages of the combined air-artillery attack system, we are now able to consider recommendations for further analysis. These recommendations for further study include the development of more general models in which gaussian lethality functions are applied to the air attack system and to the artillery

system. This would allow the assessment of the combined air-artillery attack system against a full spectrum of actual target types. The effects of target location uncertainty could also be included in this study.

Another possible area that would be valuable to study would be the application of the combined air-artillery system against a large area target, such as a mixed armor-infantry attack involving battalion size units.

Trade off implications of the combined air-artillery technique for a pure artillery attack or a pure air attack in a scenario in which we are limited in available supporting arms resources could be examined.

A multiple-gun artillery attack involving "closed sheaf" volley fire in combination with multiple bombs dropped on each attack aircraft pass could also be studied. Additionally, this study should include the use of multiple-gun artillery attacks from different firing sites in conjunction with the air attack system.

A statistical analysis of the precision distruction mission for the purpose of estimating the correlation between rounds for various gun-target ranges, plus a statistical analysis involving least squares regression of CEP as a function of release altitude, and a statistical analysis of an air attack against a point target to determine if there is any correlation between passes (for the same aircraft or flight of aircraft) would be valuable.

It is hoped that the analysis of this supporting arms application technique will be of assistance to future tacticians, and will generate an interest in future investigations of supporting arms mixes and tactics.

SUMMARY OF RESULTS

The purpose of this study was to assess the feasibility of simultaneously using close air support attack and artillery in attacking a target. The feasibility of this tactic is clearly demonstrated if certain restrictions are imposed on the artillery and air attack systems. The artillery restrictions involve limiting angles of fire to those which result in angles of impact of 533 mils (30°) or less and the exclusion of V-T fuzed projectiles. A "Danger Air Space" is specified which includes almost all of the possible artillery trajectories in attacking the target. From the geometric implications of the Danger Air Space, restrictions for the attacking aircraft are then established. These restrictions are based upon three assumptions, that aircraft pull-up begins no later than 1000 feet below the bomb release point (1.9 seconds after bomb release for an aircraft diving at 45° and moving at 450 knots, or 2.6 seconds for an aircraft diving at 30° and moving at 450 knots), that the aircraft executes a minimum 3-G pull-up, and that the attack aircraft does not exceed the allowable attack azimuth restrictions by more than 15°. From these assumptions it was determined that for bombs released at 3000 feet from a 45° dive, the safe attack azimuths were $\pm 60^{\circ}$ relative to the artillery gun-target line. The safe attack azimuths for a 45° dive and bomb release at 5000 feet are $\pm 135^{\circ}$ relative to

the artillery gun-target line. Similar azimuth restrictions were developed for other modes of aircraft delivery.

Some of the disadvantages of the simultaneous air-artillery attack technique are:

1) the exclusion of V-T fuzed artillery projectiles,

2) the maximum allowable gun-target range for the 155mm Howitzer of 12,500 meters (charge 8),

3) the exclusion of high angle fire for the artillery,

4) the limitations on the attack azimuths of the strike aircraft,

5) the possible feelings of apprehension on the pilot's part due to the proximate artillery shells,

6) the requirement of close cooperation between infantry, air, and artillery in implimentation of the technique, and

7) the artillery must be in, or ready for, fire-for-effect before the air strikes begin.

Some advantages of the simultaneous use of air and artillery in attacking a target are:

1) once the attack begins, it is not interrupted until the target is destroyed,

2) target aquisition by the air strike force is enhanced,

3) the reduction in time to destroy the target due to the increased rate of fire over conventional methods results in fewer sorties required to destroy the target, less exposure time to both the infantry and the attack aircraft (due to the more rapid destruction of the target),

4) other artillery units or guns are free to attack other targets that may become apparent within the Danger Air Space envelope, during the air strikes, and

5) time-on-target techniques involving TPQ-27 radar directed air strikes and artillery are feasible (when compared to normal pure artillery time-on-target missions, this mixed air-artillery time-on-target mission can result in greater shock, larger target area coverage, and a greater potential of causing damage to enemy units without loss of surprise.).

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The purpose of this report is to assess the feasibility of attacking a target with the simultaneous use of air and artillery. A method for generating circular error probability as a function of release altitude is presented. Techniques for determining probabilities of kill for the air

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attack system, artillery system, and for the combined air-artillery attack system are examined. From the probability of kill information and from the rate of fire (delivery) of the systems, expected time to target destruction calculations are developed. The restrictions that allow the use of the combined air-artillery attack system are presented, as well as a discussion of the advantages and disadvantages of this system of attack.

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